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Suggested Solution for Pre-lecture Problems for Lecture 3

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1. (a) The standard form is

$$\begin{aligned} \max \quad & 5x_1 + 3x_2 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 = 16 \\ & x_1 + 4x_2 + x_4 = 20 \\ & x_2 + x_5 = 8 \\ & x_i \geq 0 \quad \forall i = 1, ..., 5. \end{aligned}$$

(b) Since in the standard form we have five variables and three constraints, there should be three basic variables and two nonbasic variables in a basic solution. The ten possible ways to choose two (nonbasic) variables to be 0 are listed in the table below. The basic feasible solutions are $(\frac{44}{3}, \frac{4}{3}, 0, 0, \frac{20}{3})$, $(16, 0, 0, 4, 8)$, $(0, 5, 11, 0, 3)$, and $(0, 0, 16, 20, 8)$.

x_1	x_2	x_3	x_4	x_5	basis
-12	8	20	0	0	$\{x_1, x_2, x_3\}$
8	8	0	-20	0	$\{x_1, x_2, x_4\}$
$\frac{44}{3}$	$\frac{4}{3}$	0	0	$\frac{20}{3}$	$\{x_1, x_2, x_5\}$
N/A	0	N/A	N/A	0	$\{x_1, x_3, x_4\}$
20	0	-4	0	8	$\{x_1, x_3, x_5\}$
16	0	0	4	8	$\{x_1, x_4, x_5\}$
0	8	8	-12	0	$\{x_2, x_3, x_4\}$
0	5	11	0	3	$\{x_2, x_3, x_5\}$
0	16	0	-44	-8	$\{x_2, x_4, x_5\}$
0	0	16	20	8	$\{x_3, x_4, x_5\}$

(c) The one-to-one mapping between bfs and extreme points is shown in Figure 1.

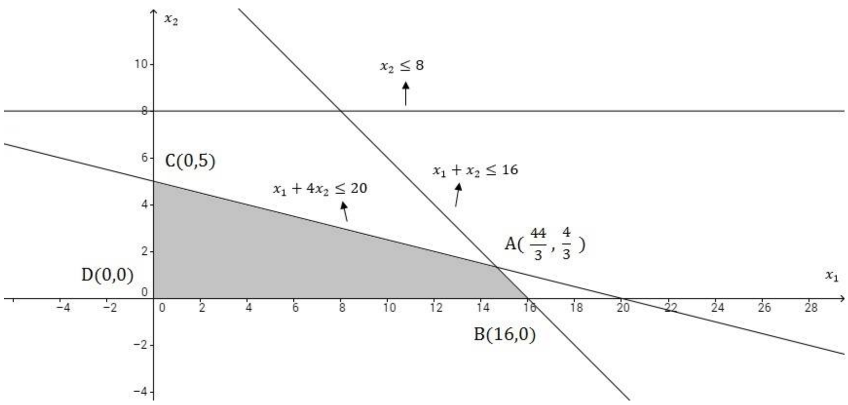

Figure 1: Graphical solution for Problem 1c

2. The initial tableau is

-5	-3	0	0	0	0
1	1	1	1	0	$x_3 = 16$
1	4	0	0	1	$x_4 = 20$
0	1	0	0	1	$x_5 = 8$

We run two iterations to get

-5	-3	0	0	0	0
$\boxed{1}$	1	1	1	0	$x_3 = 16$
1	4	0	0	1	$x_4 = 20$
0	1	0	0	1	$x_5 = 8$

$\rightarrow$

0	2	5	0	0	80
1	1	1	0	0	$x_1 = 16$
0	3	-1	1	0	$x_4 = 4$
0	1	0	0	1	$x_5 = 8$

An optimal solution to the original LP is $(x_1^*, x_2^*) = (16, 0)$ with objective value $z^* = 80$.

3. (a) Let x_1 and x_2 be the number of tables and chairs produced, respectively. The standard form is

$$\begin{aligned} \max \quad & 120x_1 + 80x_2 - 30(3x_1 + 2x_2) \\ \text{s.t.} \quad & 3x_1 + 2x_2 + x_3 = 10 \\ & \frac{1}{0.5}x_1 + x_2 + x_4 = 12 \\ & x_i \geq 0 \quad \forall i = 1, ..., 4. \end{aligned}$$

The bfs are as below:

x_1	x_2	x_3	x_4	basis
$\frac{10}{3}$	0	0	$\frac{16}{3}$	$\{x_1, x_4\}$
0	5	0	7	$\{x_2, x_4\}$
0	0	10	12	$\{x_3, x_4\}$

(b) The initial tableau is

-30	-20	0	0	0
3	2	1	0	$x_3 = 10$
2	1	0	1	$x_4 = 12$

We run two iterations to get

-30	-20	0	0	0
$\boxed{3}$	2	1	0	$x_3 = 10$
2	1	0	1	$x_4 = 12$

$\rightarrow$

0	0	10	0	100
1	$\frac{2}{3}$	$\frac{1}{3}$	0	$x_1 = \frac{10}{3}$
0	$-\frac{1}{3}$	$-\frac{2}{3}$	1	$x_4 = \frac{16}{3}$

An optimal solution to the original LP is $(x_1^*, x_2^*) = (\frac{10}{3}, 0)$ with objective value $z^* = 100$.