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Suggested Solution For Case 1

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1. Let the decision variables be

$$x_{ij} = \text{the number of products transferred from DC at location } i \text{ to store at location } j.$$

Let the parameters be

$$\begin{aligned} S_j &= \text{the number of products demanded by store at location } j, \\ M_i &= \text{cost per product of maintaining the operation in DC at location } i, \\ K_i &= \text{maximum scale of DC at location } i, \text{ and} \\ D_{ij} &= \text{distant between DC at location } i \text{ and store at location } j. \end{aligned}$$

The formulation is

$$\begin{aligned} \min \quad & \sum_{i=1}^{10} \left(M_i \sum_{j=1}^{100} x_{ij} \right) + \sum_{i=1}^{10} \sum_{j=1}^{100} D_{ij} x_{ij} \\ \text{s.t.} \quad & \sum_{i=1}^{10} x_{ij} = S_j \quad \forall j = 1, ..., 100 \\ & \sum_{j=1}^{100} x_{ij} \leq K_i \quad \forall i = 1, ..., 10 \\ & x_{ij} \geq 0 \quad \forall i = 1, ..., 10 \quad \forall j = 1, ..., 100. \end{aligned}$$

- Objective function: Adding maintenance cost(the former one) and replenishment cost(the latter one).
- Constraint 1: All the demand of each store must be exactly satisfied.
- Constraint 2: The scale of each DC cannot exceed its maximum scale.
- Constraint 3: The number of products transferred must be nonnegative.

2. Check the AMPL model "case1.mod" and data "case1.dat". After solving, the minimum cost is 143165 and the construction cost is 24150. Thus, we need 167315 in total cost.

3. Table 1 is the optimal scale of each DC.

DC	1	4	6	8	9	10
Total	1491	550	1077	361	589	404

Table 1: optimal scale of each DC

Table 2 is the replenishment plan

S \ D	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	53	0	0	0	0
2	0	0	0	0	0	79	0	0	0	0
3	0	0	0	0	0	34	0	0	0	0
4	0	0	0	0	0	29	0	0	0	0
5	0	0	0	0	0	48	0	0	0	0

6	0	0	0	0	0	0	0	51	0	0
7	0	0	0	0	0	0	0	0	0	34
8	0	0	0	0	0	11	0	0	0	0
9	0	0	0	0	0	0	0	0	96	0
10	40	0	0	0	0	0	0	0	0	0
11	13	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	19
13	0	0	0	0	0	0	0	0	0	5
14	0	0	0	90	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	4	0
16	0	0	0	86	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	81
18	0	0	0	0	0	0	0	86	0	0
19	0	0	0	0	0	49	0	0	0	0
20	0	0	0	0	0	0	0	0	29	0
21	0	0	0	0	0	0	0	0	0	92
22	54	0	0	0	0	0	0	0	0	0
23	14	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	63	0	0
25	0	0	0	0	0	54	0	0	0	0
26	0	0	0	0	0	0	0	0	56	0
27	0	0	0	0	0	90	0	0	0	0
28	0	0	0	0	0	22	0	0	0	0
29	0	0	0	1	0	0	0	0	0	0
30	0	0	0	0	0	27	0	0	0	0
31	35	0	0	0	0	0	0	0	0	0
32	0	0	0	89	0	0	0	0	0	0
33	36	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	32	0	0	0	0
35	20	0	0	0	0	0	0	0	0	0
36	0	0	0	41	0	0	0	0	0	0
37	26	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	17	0
39	0	0	0	0	0	0	0	0	0	67
40	0	0	0	0	0	0	0	0	0	49
41	0	0	0	0	0	54	0	0	0	0
42	0	0	0	28	0	0	0	0	0	0
43	0	0	0	76	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	93	0
45	0	0	0	0	0	79	0	0	0	0
46	0	0	0	0	0	0	0	0	91	0
47	0	0	0	0	0	0	0	44	0	0
48	47	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	47	0
50	0	0	0	7	0	0	0	0	0	0
51	0	0	0	0	0	52	0	0	0	0
52	0	0	0	0	0	0	0	0	8	0
53	0	0	0	0	0	28	0	0	0	0
54	87	0	0	0	0	0	0	0	0	0
55	0	0	0	46	0	0	0	0	0	0
56	0	0	0	0	0	0	0	0	0	25
57	0	0	0	0	0	66	0	0	0	0
58	43	0	0	0	0	0	0	0	0	0
59	14	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	88	0	0	0	0
61	0	0	0	0	0	0	0	0	22	0

62	0	0	0	0	0	0	0	0	62	0
63	15	0	0	0	0	0	0	0	0	0
64	17	0	0	0	0	0	0	0	0	0
65	77	0	0	0	0	0	0	0	0	0
66	78	0	0	0	0	0	0	0	0	0
67	0	0	0	0	0	50	0	0	0	0
68	16	0	0	0	0	0	0	0	0	0
69	0	0	0	34	0	0	0	0	0	0
70	95	0	0	0	0	0	0	0	0	0
71	44	0	0	0	0	0	0	0	0	0
72	53	0	0	0	0	0	0	0	0	0
73	41	0	0	0	0	0	0	0	0	0
74	0	0	0	0	0	0	0	0	7	0
75	0	0	0	0	0	0	0	23	0	0
76	28	0	0	0	0	0	0	0	0	0
77	0	0	0	0	0	0	0	0	0	7
78	57	0	0	0	0	0	0	0	0	0
79	58	0	0	0	0	0	0	0	0	0
80	99	0	0	0	0	0	0	0	0	0
81	51	0	0	0	0	0	0	0	0	0
82	0	0	0	0	0	28	0	0	0	0
83	0	0	0	0	0	13	0	0	0	0
84	0	0	0	0	0	0	0	0	0	25
85	0	0	0	0	0	43	0	0	0	0
86	19	0	0	0	0	0	0	0	0	0
87	0	0	0	0	0	0	0	0	17	0
88	0	0	0	52	0	0	0	0	0	0
89	0	0	0	0	0	0	0	0	33	0
90	0	0	0	0	0	0	0	79	0	0
91	89	0	0	0	0	0	0	0	0	0
92	59	0	0	0	0	0	0	0	0	0
93	0	0	0	0	0	3	0	0	0	0
94	0	0	0	0	0	45	0	0	0	0
95	7	0	0	0	0	0	0	0	0	0
96	0	0	0	0	0	0	0	0	7	0
97	1	0	0	0	0	0	0	0	0	0
98	58	0	0	0	0	0	0	0	0	0
99	0	0	0	0	0	0	0	15	0	0
100	100	0	0	0	0	0	0	0	0	0
Total	1491	0	0	550	0	1077	0	361	589	404

Table 2: Replenishment plan

Figure 1 is the scatter plot of the replenishment plan.

4. (a) We can decide whether it is worthwhile to expand its maximum scale by the shadow prices of maximum scale constraints. According to the calculation, we know that all the shadow prices of maximum scale constraints are 0, which indicates that no maximum scale should be expanded.

   (b) To determine where to put more marketing budget to boost demands, we should calculate shadow price of each store's demand. The shadow prices are shown in Figure 2. According to Figure 2, all the shadow prices of each demand are positive. That is, when there is one unit of demand increase, the total cost would become higher. Then we should go a step further to consider the unit price information of meat in each store.We can split the problem into two parts.

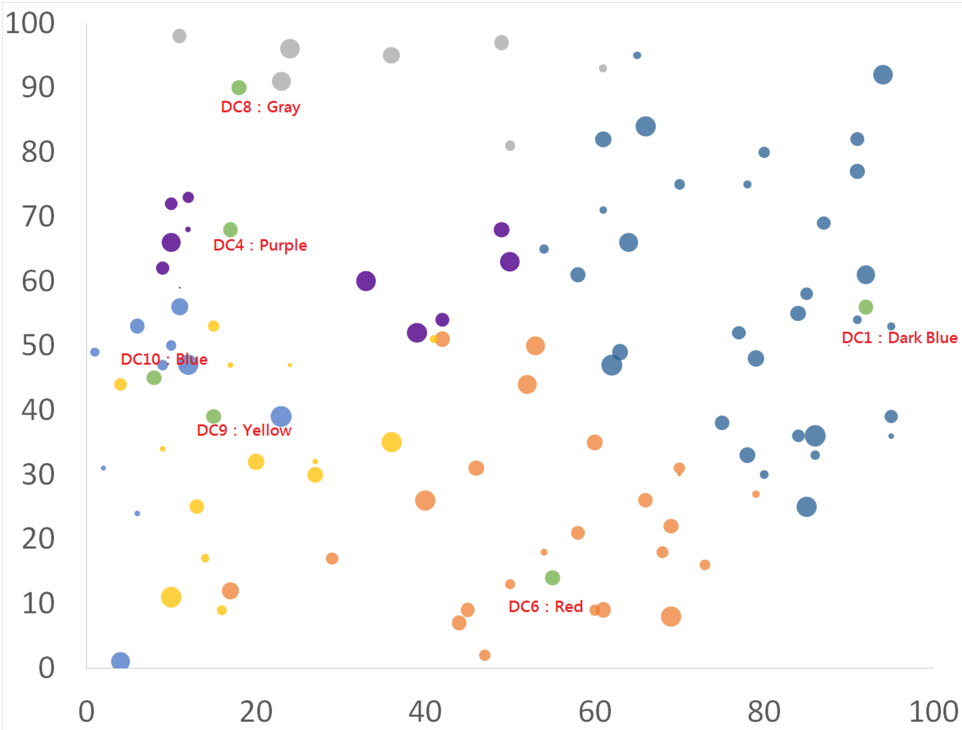

Figure 1: Replenishment plan.

```
1 : 12  13: 32  25: 27  37: 43  49; 29  61: 44  73: 22  85: 11  97 : 10
2 : 39  14: 36  26: 34  38: 51  50: 17  62: 25  74: 23  86: 31  98 : 59
3 : 30  15: 30  27: 28  39: 26  51: 51  63:  8  75: 57  87: 36  99 : 62
4 : 33  16: 50  28:  7  40: 22  52: 24  64:  5  76: 38  88: 44  100: 28
5 : 24  17: 60  29: 27  41: 27  53: 21  65:  7  77: 35  89: 31
6 : 54  18: 28  30: 21  42: 22  54: 40  66: 40  78: 38  90: 22
7 : 17  19: 19  31: 11  43: 21  55: 51  67: 23  79: 39  91: 40
8 :  6  20: 27  32: 50  44: 46  56: 19  68: 40  80: 41  92: 23
9 : 21  21: 18  33: 30  45: 34  57: 41  69: 23  81: 41  93: 32
10: 21  22: 11  34: 18  46: 38  58: 20  70: 56  82: 11  94: 16
11: 68  23: 48  35: 49  47: 31  59: 35  71: 29  83: 38  95: 25
12: 23  24: 39  36: 26  48: 37  60: 21  72: 24  84: 15  96: 32
```

Figure 2: Shadow price of each demand

i. If there are price information for each store, we can get the profit by subtracting replenishment and maintaince cost from revenue. If the profit of the store become higher when one unit of the store demand increase, then we should put more marketing budget on that store. Otherwise, we shouldn't do it.

ii. If there is no price information for each store, then we should put marketing budget on three kinds of store.

A. The store with the shortest distance from DC.

B. The store with lowest maintaince cost DC.

C. The store with the shortest distance from DC and with lowest maintaince cost DC.

5. There are two methods.

(a) The first method is

i. Pick one DC with the lowest construction cost and check if it can meet the demand. If it can, put it in set S and stop the algorithm. Otherwise, continue picking DCs into set S according to the construction cost until all demands are fulfilled. Calculate the total cost c_i according to the DCs in set S.

ii. Exchange the highest-maintenance-cost DC in set S with the lowest-maintenance-cost DC which is not in set S, and calculate the total cost c_j according to DCs in set S. If c_j

is higher than c_i, stop the algorithm. Otherwise, repeat the exchange process until c_j is lower than c_i.

iii. Output set S.

(b) The second method is

i. We solve the problem with a model below

```
param D; #10
param S; #100
param Maintenance{i in 1..D}; #Maintenance cost
param Demand{j in 1..S};
param Scale{j in 1..S};
param ConstructionCost{i in 1..D};
param Distance{i in 1..D, j in 1..S};

var x{i in 1..D, j in 1..S}; #Replenishment plan
var Settle{i in 1..D};

minimize cost:
sum{i in 1..D}( Maintenance[i] * sum{j in 1..S}(x[i, j]))+
sum{i in 1..D, j in 1..S} (Distance[i, j] * x[i, j])+
sum{i in 1..D}(ConstructionCost[i] * Settle[i]);
#Maintenance cost + Replenishment cost + Construction Cost

subject to demandConstraint{j in 1..S}: #Demand = Replenishment plan
    sum{i in 1..D}(x[i, j]) = Demand[j];
subject to ScaleConstraint{i in 1..D}: #Scale limitation
    sum{j in 1..S}(x[i, j]) <=Settle[i]*Scale[i];
subject to nonnegX{i in 1..D, j in 1..S}: #non negative
    x[i, j] >= 0;
```

ii. The solution is shown in Table 3.

Settle[i]	value
1	0.820857
2	0
3	0.244624
4	0.476923
5	0
6	0.823439
7	0.530435
8	0.127419
9	0.843284
10	0.562421

Table 3: Solution of our model

iii. We cannot get binary answers to Settle variable by solving the LP. However, the up coming chapter is about IP and we will learn more about it.