Operations Research, Spring 2017
Homework 1

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1 Problems

1. (20 points; 5 points each) Consider the following LP

$$\begin{aligned} \max \quad & 2x_1 + x_2 \\ \text{s.t.} \quad & x_1 - x_2 \leq 4 \\ & x_1 + x_2 \leq 8 \\ & x_1 \leq 5 \\ & x_1 \geq 0, x_2 \text{ urs.} \end{aligned}$$

   (a) Graphically solve the LP.

   (b) Find all the basic feasible solutions of the standard form LP.

   (c) Use the simplex method with the smallest index rule to solve the LP. Write down all the derivations and then clearly indicate your conclusion.

   (d) Change the objective function to max $x_1 - 2x_2$. Use the simplex method with the smallest index rule to solve the LP.

2. (15 points) Consider the following LP

$$\begin{aligned} \max \quad & x_1 + 3x_2 + 2x_3 \\ \text{s.t.} \quad & x_1 + x_2 \leq 4 \\ & x_1 + 2x_2 + 3x_3 \geq 10 \\ & x_3 \geq 3 \\ & x_i \geq 0 \quad \forall i = 1, ..., 3. \end{aligned}$$

   (a) (10 points) Use the simplex method with the smallest index rule to solve it. Write down all the derivations and then clearly indicate your conclusion.

   (b) (5 points) Use AMPL to solve this problem. Submit your model file (and data file, if any) as your answer.

3. (20 points; 5 point each) Please answer the following true/false questions. If your answer is "true," briefly explain why; otherwise, provide a counterexample.

   (a) An LP's optimal solution is always an extreme point.

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(b) For a standard form LP with n variables and m equality constraints, it is possible to have fewer than $\binom{n}{m}$ bases.

(c) For a minimization LP, we may conclude that it is unbounded if in a tableau we find a column with (1) a positive reduced cost and (2) all numbers in rows 1 to m are nonpositive.

(d) When we use the simplex method to solve an LP, if we use the smallest index rule, we will reach an optimal solution with the least number of iterations.

4. (45 points; 15 points each) For the coming IM Week, the student association of the IM department (STIM) needs to assign students to work in the eight time slots of five days. The time slots 9-10, 10-11, ..., and 4-5 are labeled as slots 1, 2, ..., and 8. Monday, Tuesday, ..., and Friday are labeled as days 1, 2, ..., and 5. The number of students that are needed for slot i in day j is D_{ij}. A student may work for at most sixteen slots. By considering the constraints specified in the following problems, please formulate a linear program that may help the leader of STIM to find a schedule that minimizes the number of students that are needed to fulfill all the demands.

Note 1. Please answer Parts (a) and (b) separately.

Note 2. Your program must be linear. It cannot contain nonlinear constraints or integer variables. However, the solution of your program needs not to contain only integer values. A program is helpful as long as it provides a suggestion to the decision maker, even if the suggestion needs to be manually adjusted into a feasible plan.

(a) When one works in a day, she/he must work for the all eight time slots. In other words, if she/he is going to work for sixteen hours, she/he must work in exactly two days.

(b) When one works in a day, she/he must work in either slots 1 to 4 or slots 5 to 8. In other words, she/he must work for exactly four hours in a day if she/he works in that day, and she/he must start to work at slot 1 or 5 for four consecutive slots.

(c) Suppose that the demands are given in the following table:

Slot	Number of students needed				
	Day 1	Day 2	Day 3	Day 4	Day 5
1	6	4	4	5	4
2	3	2	1	3	4
3	3	2	1	4	3
4	8	7	4	9	5
5	8	6	6	8	3
6	4	2	2	2	3
7	3	2	2	2	5
8	2	1	1	2	8

Please use AMPL to solve the problem with the constraint specified in Part (b). Copy and paste your model and data files and submit them as part of your answer. Based on the suggestion given by your program, please write down the final way of scheduling students (which now must be feasible) and tell us how many students you need. If you write everything into a model file, you will get no point. If you say you want to assign, e.g., 3.8 students to work on Monday morning, you will also get no point.

2 Submission rules

The deadline of this homework is 2pm, March 20, 2016. Please put a hard copy of the work into the instructor's mailbox on the first floor of the Management Building 2 by the due time. Works submitted between 2pm and 3pm will get 10 points deducted as a penalty. Submissions later than 3pm will not be accepted. Each student must submit her/his individual work.