Operations Research, Spring 2017
Suggested Solution for Pre-lecture Problems for Lecture 7

Solution providers: Share Lin
Department of Information Management
National Taiwan University

1. (a) Let the demand set I and the location set J both be the six towns , who locate at {(0, 60), (20, 50), (30, 20), (40, 80), (50, 50), (90, 60)}.

   Suppose that there are 52 weeks in a year, the 5-year demand size is $h_i \in$ {2600000, 3900000, 3120000, 8000, 2080000, 780000}, at demand $i \in I$.

   The distance between location $j \in J$ and demand $i \in I$ is d_{ij}.

   The fixed construction cost at location $j \in J$ is $f_j \in \{200000, 180000, 160000, 190000, 150000, 200000\}$.

   Let $x_j = 1$ if a DC is built at location $j \in J$ or 0 otherwise.

   Let $y_{ij} = 1$ is demand $i \in I$ is served by DC at location $j \in J$ or 0 otherwise.

$$
\begin{aligned}
\min \quad & \sum_{i \in I} \sum_{j \in J} \frac{h_i}{500} d_{ij} y_{ij} + \sum_{j \in J} f_j x_j \\
\text{s.t.} \quad & y_{ij} \leq x_j \quad \forall i \in I, j \in J \\
& \sum_{j \in J} y_{ij} = 1 \quad \forall i \in I \\
& x_3 = 1 \\
& x_j \in \{0, 1\} \quad \forall j \in J \\
& y_i \in \{0, 1\} \quad \forall i \in I, j \in J.
\end{aligned}
$$

   (b) Reset the location set J to {(0, 60), (20, 50), (30, 20), (40, 80), (50, 50), (90, 60), (0, 20), (20, 40), (40, 30), (60, 40)} and everything follows.

2. (a) For each job $j \in J = \{1, 2, ..., 10\}$, let the processing time p_j and due time d_j, where $p_j \in$ {6, 9, 3, 5, 10, 6, 3, 9, 7, 10} and $d_j \in$ {50, 53, 55, 56, 59, 60, 62, 67, 68,70}.

   let C_j be the completion time of job $j \in J$ and T_j be the tardiness of job $j \in J$.

   Let $z_{ij} = 1$ if job j is before job i or 0 otherwise, $i \in J, j \in J, i < j$.

   Let $M = \sum_{j \in J} p_j$.

$$
\begin{aligned}
\min \quad & \sum_{j \in J} T_j \\
\text{s.t.} \quad & T_j \geq C_j - d_j && \forall j \in J \\
& C_i + p_j - C_j \leq M z_{ij} && \forall i \in J, j \in J, i < j \\
& C_j + p_i - C_i \leq M(1 - z_{ij}) && \forall i \in J, j \in J, i < j \\
& T_j \geq 0, C_j \geq 0 && \forall j \in J \\
& z_{ij} \in \{0, 1\} && \forall i \in J, j \in J, i < j.
\end{aligned}
$$

(b)

$$\begin{array}{lll}
\min & \sum_{j \in J} T_j & \\
\text{s.t.} & T_j \geq C_j - d_j & \forall j \in J \\
& C_i + p_j - C_j \leq M z_{ij} & \forall i \in J, j \in J, i < j \\
& C_j + p_i - C_i \leq M(1 - z_{ij}) & \forall i \in J, j \in J, i < j \\
& T_j \geq 0, C_j \geq 0 & \forall j \in J \\
& z_{15} = 0 & \\
& z_{34} = 0 & \\
& z_{57} = 0 & \\
& z_{67} = 0 & \\
& z_{69} = 0 & \\
& z_{6(10)} = 0 & \\
& z_{ij} \in \{0, 1\} & \forall i \in J, j \in J, i < j.
\end{array}$$

3. (a) For each job $j \in J = \{1, 2, ..., 10\}$, let the processing time p_j, due time d_j, and release time r_j, where $p_j \in$ {6, 9, 3, 5, 10, 6, 3, 9, 7, 10}, $d_j \in$ {50, 53, 55, 56, 59, 60, 62, 67, 68,70}, and $r_j \in$ {8, 0, 2, 14, 12, 4, 20, 22, 18, 15}.

let C_j be the completion time of job $j \in J$ and T_j be the tardiness of job $j \in J$.

Let $z_{ij} = 1$ if job j is before job i or 0 otherwise, $i \in J, j \in J, i < j$.

Let $M = \sum_{j \in J} p_j$.

$$\begin{array}{lll}
\min & \sum_{j \in J} T_j & \\
\text{s.t.} & T_j \geq C_j - d_j & \forall j \in J \\
& C_i + p_j - C_j \leq M z_{ij} & \forall i \in J, j \in J, i < j \\
& C_j + p_i - C_i \leq M(1 - z_{ij}) & \forall i \in J, j \in J, i < j \\
& C_j \geq r_j + p_j & \forall j \in J \\
& T_j \geq 0, C_j \geq 0 & \forall j \in J \\
& z_{ij} \in \{0, 1\} & \forall i \in J, j \in J, i < j.
\end{array}$$

(b) The optimal schedule is {2, 3, 1, 6, 4, 5, 7, 8, 9, 10}, with completion time {9, 12, 18, 24, 29, 39, 42, 51, 58, 68} and total tardiness 0.