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Suggested Solution for Midterm Exam

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1. (a) The manufacturer solves $\pi_M =$

$$\begin{aligned} \max \quad & Qw - \frac{1}{2}cQ^2 \\ \text{s.t.} \quad & Q \leq q \end{aligned}$$

and obtains $Q^* = \min\{\frac{w}{c}, q\}$.

(b) The retailer solves $\pi_R =$

$$\max \quad \int_0^q xf(x)dx + \int_q^1 qf(x)dx - wq$$

and obtains $q^* = 1 - w$.

(c) When $1 - w \leq \frac{w}{c}$, $Q^* = q^* = 1 - w$. Plug Q^* into π_M, solve it, and we obatin $w_1 = \frac{1+c}{2+c}$. When $1 - w \geq \frac{w}{c}$, $Q^* = \frac{w}{c}$. Plug Q^* into π_M, solve it, and we obtain $w_2 = \frac{c}{1+c}$. Now, we replace w in π_M by w_1 or w_2, and find out that $w^* = w_1 = \frac{1+c}{2+c}$ is the optimal wholesale price.

(d) When the wholesale contract may lead to low order quantity (e.g., when the retail price is too low) and the production cost is small enough, the manufacturer would offer a positive return credit in equilibrium.

2. (a) Under integration, $p_1 = p_2 = p$. Maximize $pq = p(1 - p + \theta p)$ and we can obtain $p^* = \frac{1}{2-2\theta}$.

(b) Maximize $p_i q_i = p_i(1 - p_i + \theta p_{3-i})$ for $i = 1, 2$ and we can obtain $p_1^* = p_2^* = \frac{1}{2-\theta}$.

(c) Maximize $(p_1 - w_1)(1 - p_1 + \theta p_2) + (p_2 - w_2)(1 - p_2 + \theta p_1)$ and we can obtain $p_i^* = \frac{1}{2-2\theta} + \frac{w_i}{2}$ for $i = 1, 2$.

(d) Maximize $w_i(1 - (\frac{1}{2-2\theta} + \frac{w_i}{2}) + \theta(\frac{1}{2-2\theta} + \frac{w_{3-i}}{2}))$ for $i = 1, 2$, and we can obtain $w_i^* = \frac{1}{2-\theta}$.

(e) Under ID, the profit function for the retailer and the firms would be:

$$\pi_R = (p_1 - w_1)(1 - p_1 + \theta p_2) \qquad \pi_2^M = p_2(1 - p_2 + \theta p_1) \qquad \pi_1^M = w1(1 - p_1 + \theta p_2).$$

First, we may find the optimal p_1^* and p_2^* by solving the F.O.C. of π_R and π_2^M:

$$\frac{\partial \pi_R}{\partial p_1} = 1 - 2p_1 + \theta p_2 + w_1 = 0 \qquad \frac{\partial \pi_2^M}{\partial p_2} = 1 - 2p_2 + \theta p_1 = 0$$

$$p_1^* = \frac{1}{2-\theta} + \frac{2w_1}{4-\theta^2} \qquad p_2^* = \frac{1}{2-\theta} + \frac{\theta w_1}{4-\theta^2}.$$

Then, we can plug in p_1^* and p_2^* to π^M, solve the F.O.C., and obtain $w_1^* = \frac{2+\theta}{4-2\theta^2}$.

(f) True. The equilibruim retail prices under II is $p_i = \frac{1}{2-2\theta} + \frac{w_i}{2}$ for $i = 1, 2$, which is greater than prices under pure integration ($p_1 = p_2 = \frac{1}{2-2\theta}$).

(g) True. The equilibruim retail prices under II is $p_i = \frac{1}{2-2\theta} + \frac{w_i}{2}$ for $i = 1, 2$, which is greater than prices under DD ($p_1 = p_2 = \frac{1}{2-\theta}$).

(h) True. p_1 under ID is $\frac{1}{2-\theta} + \frac{2w_1}{4-\theta^2}$, which is greater than p_1 under DD ($p_1 = \frac{1}{2-\theta}$). p_2 under ID is $\frac{1}{2-\theta} + \frac{\theta w_1}{4-\theta^2}$, which is also greater than p_2 under DD ($p_2 = \frac{1}{2-\theta}$).

3. (a) $P_{LB} = Pr(\theta = \theta_L | s = s_B) = \frac{Pr(s=s_B \cap \theta=\theta_L)}{Pr(s=s_B)} = \frac{Pr(s=s_B|\theta=\theta_L)Pr(\theta=\theta_L)}{Pr(s=s_B)} = \frac{\lambda * \frac{1}{2}}{\frac{1}{2}} = \lambda$.

(b) $N_B = E[\theta|s = s_B] = Pr(\theta = \theta_L|s = s_B)\theta_L + Pr(\theta = \theta_H|s = s_B)\theta_H = \lambda\theta_L + (1-\lambda)\theta_H$.

(c) $Q_B = Pr(s = s_B) = \frac{1}{2}$.

(d) First, we may define π_S as salesperson's utility function,

$$\pi_S = \max_{a_{jk}} \mathbb{E}[u_k + v_k x - \frac{1}{2}a_{jk}^2|s = s_j] = \max_{a_{jk}}(u_k + v_k N_j a_{jk} - \frac{1}{2}a_{jk}^2).$$

We then have the F.O.C.

$$\pi_S' = -a_{jk} + v_k N_j = 0.$$

After solving the F.O.C., we have the optimal effort

$$a_{jk}^* = v_k N_j.$$

(e) The retailer's objective function can be formulated as

$$\pi_R = \max_{u_k urs., v_k \geq 0} \sum_{j\in\{G,B\}} \frac{1}{2}[(1-v_k)N_j^2 v_k - u_k].$$

(f) From (d), the salesperson's optimal profit can be formulated as

$$\pi_S^* = u_k + \frac{1}{2}v_k^2 N_j^2.$$

The IR constraints can then be formulated as,

$$u_G + \frac{1}{2}v_G^2 N_G^2 \geq 0, \quad \text{(IR-1)}$$

$$u_B + \frac{1}{2}v_B^2 N_B^2 \geq 0. \quad \text{(IR-2)}$$

The IC constraints can then be formulated as,

$$u_G + \frac{1}{2}v_G^2 N_G^2 \geq u_B + \frac{1}{2}v_B^2 N_G^2, \quad \text{(IC-1)}$$

$$u_B + \frac{1}{2}v_B^2 N_B^2 \geq u_G + \frac{1}{2}v_G^2 N_B^2. \quad \text{(IC-2)}$$

(g) Let (IR-2) and (IC-1) bind, we have

$$u_B = -\frac{1}{2}v_B^2 N_B^2,$$

$$u_G = -\frac{1}{2}v_B^2 N_B^2 + \frac{1}{2}v_B^2 N_G^2 - \frac{1}{2}v_G^2 N_G^2.$$

Plug them into the retailer's objective function, we have

$$\pi_R = \frac{1}{2}[(1-v_G)N_G^2 v_G + (1-v_B)N_B^2 v_B + \frac{1}{2}v_B^2 N_B^2 + \frac{1}{2}v_B^2 N_B^2 - \frac{1}{2}v_B^2 N_G^2 + \frac{1}{2}v_G^2 N_G^2]$$
$$= \frac{1}{2}[N_G^2 v_G + N_B^2 v_B - \frac{1}{2}N_G^2 v_G^2 - \frac{1}{2}N_G^2 v_B^2].$$

Next, we have the partial differential equations

$$\frac{\partial \pi_R}{\partial v_G} = \frac{1}{2}[N_G^2 - N_G^2 v_G] = 0,$$

$$\frac{\partial \pi_R}{\partial v_B} = \frac{1}{2}[N_B^2 - N_G^2 v_B] = 0.$$

Finally, we have the optimal v_k: $v_G^* = 1; v_B^* = \frac{N_B^2}{N_G^2}$,
and the optimal u_k: $u_G^* = \frac{1}{2}(-\frac{N_B^6}{N_G^4} + \frac{N_B^4 N_G^2}{N_G^4} - N_G^2); u_B^* = -\frac{1}{2}\frac{N_B^6}{N_G^4}$.

(h) Step 1:

$$\begin{aligned}
\bar{a} &= \sum_{j\in\{G,B\}} Pr(s = s_j)a^*_{jj} \\
&= \frac{1}{2}v_G N_G + \frac{1}{2}v_B N_B \\
&= \frac{1}{2}N_G + \frac{1}{2}\frac{N_B^2}{N_G^2}N_B \\
&= \frac{1}{2}\frac{N_G^3 + N_B^3}{N_G^2} \\
&= \frac{1}{2}\frac{(\theta_L(1-\lambda)+\theta_H\lambda)^3 + (\theta_L\lambda + \theta_H(1-\lambda))^3}{(\theta_L(1-\lambda)+\theta_H\lambda)^2}.
\end{aligned}$$

Step 2:

$$\begin{aligned}
\frac{\partial \bar{a}}{\partial \lambda} &= \frac{1}{2}(-\frac{(\theta_H-\theta_L)(\theta_H+\theta_L)^2(\theta_H(3\lambda-2)-\theta_L(3\lambda-1))}{(\theta_L(\lambda-1)-\theta_H\lambda)^3}) \\
&= \frac{1}{2}(-\frac{(\theta_H-\theta_L)(\theta_H+\theta_L)^2((\theta_H-\theta_L)(3\lambda-1)-\theta_H)}{(-N_G)^3}).
\end{aligned}$$

Finally, we find that when the difference between θ_H and θ_L is large enough, λ has positive effects on $\bar{a}$. In the contrary, when the difference between θ_H and θ_L is small, even if λ is large, λ still has negative effects on $\bar{a}$.