

# Information Economics, Fall 2016

## Pre-lecture Problems 6

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**Note 1.** The deadline of submitting the pre-lecture problem is **9:20am, October 24, 2016**. Please submit a hard copy of your work to the instructor in class. Late submissions will not be accepted. Each student must submit her/his individual work. Submit ONLY the problem that counts for grades.

**Note 2.** Please make your answer as clear (i.e., easy to read) as possible. We reserve the right to take away points when the correctness cannot be easily determined (e.g., when the writing is messy and cannot be easily understood).

1. (0 points) Consider the two-type model discussed in class. One key feature of this model is the IC constraint, which describes how the seller may differentiate consumers by inducing them to act differently. In particular, we have

$$\begin{aligned} \theta_H v(q_H) - T_H &\geq \theta_H v(q_L) - T_L & (\text{IC-H}) \\ \theta_L v(q_L) - T_L &\geq \theta_L v(q_H) - T_H & (\text{IC-L}) \end{aligned}$$

as our IC constraints.

- (a) Explain why  $\theta_H$  is not  $\theta_L$  at the right-hand side of (IC-H).  
 (b) Verify monotonicity, i.e.,  $q_H \geq q_L$ .
2. (0 points) Consider a set of consumers whose types  $\theta$  lie in an interval  $[0, 1]$  uniformly. Each of these consumers are considering buying a product of two versions with quality levels  $q_1$  and  $q_2$ , where  $0 < q_1 < q_2$ . A type- $\theta$  consumer's utility is  $\theta q - p$  if he buys the version of quality  $q$  by paying  $p$  to the seller. A consumer can either buy version 1, buy version 2, or buy nothing (which gives him a zero utility). He makes the decision to maximize his utility. Let the prices of the two versions be  $p_1$  and  $p_2$ , respectively, where  $p_1 < p_2$ . We assume that  $q_1 > p_1$  and  $q_2 > p_2$  so that at least the highest-type consumer is willing to buy something. We further assume that

$$\frac{p_2 - p_1}{q_2 - q_1} > \frac{p_1}{q_1}. \quad (1)$$

- (a) For a type- $\theta$  consumer, under what condition will he prefers buying version 1 to buying nothing? Is this some kind of IR constraint?  
 (b) For a type- $\theta$  consumer, under what condition will he prefers buying version 2 to version 1? Is this some kind of IC constraint?
3. (10 points) Consider the two-type model we introduced in class. Suppose that there are two consumers whose types are  $\theta_1 = 10$  and  $\theta_2 = 15$ . For a type- $\theta$  consumer, his utility is

$$\theta v(q) - T,$$

where  $q$  is his consumption level,  $T$  is the price he pays, and  $v(q) = \alpha(q - q^2)$  with some  $\alpha > 0$ . Suppose that 40% of the consumers are type- $\theta_1$  and 60% are type- $\theta_2$ . The unit production cost of the product is  $c = 50$ . The seller's utility is  $T - cq$ .

- (a) (4 points) Find the first best quantities  $\tilde{q}_1$  and  $\tilde{q}_2$ , respectively.  
 (b) (4 points) Find the second best quantities  $q_1^*$  and  $q_2^*$ , respectively.  
 (c) (2 points) Show that downward distortion exists for all values of  $\alpha$  as long as  $\tilde{q}_1 > 0$ .