

Information Economics

Introduction and Review of Optimization

Ling-Chieh Kung

Department of Information Management
National Taiwan University

Road map

- ▶ **Course overview.**
- ▶ Convexity and optimization.
- ▶ Applications.

Welcome!

- ▶ This is **Information Economics**, NOT **Information Economy**.
 - ▶ We do not put emphasis on IT, IS, information goods, etc.
 - ▶ We focus on **information**.
- ▶ We focus on the **economics of information**.
 - ▶ How people behave with different information?
 - ▶ What is the value of information?
 - ▶ What information to acquire? How?
 - ▶ What are the implications on business and economy?
- ▶ **Information asymmetry** is particularly important.

Information asymmetry

- ▶ The world is full of asymmetric information:
 - ▶ A consumer does not know a retailer's procurement cost.
 - ▶ A consumer does not know a product's quality.
 - ▶ A retailer does not know a consumer's valuation.
 - ▶ An instructor does not know how hard a student works.
- ▶ As the world is **decentralized**:
 - ▶ There is the **incentive** issue.
 - ▶ There is the **information** issue.
- ▶ As information asymmetry results in inefficiency, we want to:
 - ▶ Analyze its impact. If possible, quantify it.
 - ▶ Decide whether it introduces driving forces for some phenomena.
 - ▶ Find a way to deal with it if it cannot be eliminated.
- ▶ This field is definitely fascinating. However:
 - ▶ We need to have some "**weapons**" to explore the world!

Before you enroll...

- ▶ Prerequisites:
 - ▶ Calculus.
 - ▶ Probability.
 - ▶ Convex optimization.
 - ▶ Game theory.
- ▶ This is an **academic methodology** course.
 - ▶ It is directly helpful if you are going to write a thesis with this research methodology.
 - ▶ It can be indirectly helpful for you to analyze the real world. However, we do not train you to do that in this course.
- ▶ This course is about **science**, not **business** or **engineering**.
 - ▶ It is about **identifying reasons**.
 - ▶ It is not about **solving problems**.
 - ▶ It is not about **making decisions**.

The instructing team

- ▶ Instructor:
 - ▶ Ling-Chieh Kung.
 - ▶ Assistant professor.
 - ▶ Office: Room 413, Management Building II.
 - ▶ Office hour: by appointment.
 - ▶ E-mail: lckung@ntu.edu.tw.
- ▶ There is no teaching assistant for this course.

Related information

- ▶ Classroom: Room 204, Management Building II.
- ▶ Lecture time: 9:10am-12:10pm, Monday.
- ▶ References:
 - ▶ *Information Rules* by C. Shapiro and H. Varian.
 - ▶ *Freakonomics* by S. Levitt and S. Dubner.
 - ▶ *Contract Theory* by P. Bolton and M. Dewatripont.
 - ▶ *Game Theory for Applied Economists* by R. Gibbons.
 - ▶ About ten academic papers.

“Flipped classroom”

- ▶ Lectures in **videos**, then discussions in classes.
- ▶ Before each Monday, the instructor uploads a video of lectures.
 - ▶ Ideally, the video will be no longer than one and a half hour.
 - ▶ Students must watch the video by themselves before that Monday.
- ▶ During the lecture, we do three things:
 - ▶ Discussing the lecture materials.
 - ▶ Solving lecture problems (to earn points).
 - ▶ Further discussions.
- ▶ Teams:
 - ▶ Students form teams to work on class problems and case studies.
 - ▶ Each team should have **two or three students**.
 - ▶ Your teammates may be different from week to week.

Pre-lecture problems and class participation

- ▶ No homework!
 - ▶ Problem sets and solutions will be posted for students to do practices.
- ▶ Pre-lecture problems.
 - ▶ One problem to submit per set of lecture videos.
 - ▶ Submit a hard copy at the beginning of a lecture.
- ▶ Class participation:
 - ▶ Just say something!
 - ▶ Use whatever way to impress the instructor.

Paper presentations and projects

- ▶ Paper presentations:
 - ▶ Students will form six teams to present six academic papers. The team size will be determined according to the class size.
 - ▶ On the date that a team present, they should submit one paper summary and their slides.
- ▶ Midterm project:
 - ▶ Students form teams to do a midterm project.
 - ▶ A topic will be assigned, and each team constructs its own models and generate its own findings.
 - ▶ A written report is required.
- ▶ Final project:
 - ▶ Students form teams to do a midterm project.
 - ▶ A direction will be assigned, and each team conducts its own research by defining its own research questions.
 - ▶ Each team will submit a proposal for the self-selected topic, make a 30-minute presentation, and submit a report.

Grading

- ▶ Not dropping this course: 10%.
- ▶ Class participation: 10%.
- ▶ Pre-lecture problems: 10%.
- ▶ Paper presentations: 20%.
- ▶ Midterm project: 20%.
- ▶ Final project: 30%.
- ▶ The final letter grades will be given according to the following conversion rule:

Letter	Range	Letter	Range	Letter	Range
A+	[90, 100]	B+	[77, 80]	C+	[67, 70]
A	[85, 90)	B	[73, 77)	C	[63, 67)
A-	[80, 85)	B-	[70, 73)	C-	[60, 63)

Important dates, tentative plan, and websites

- ▶ Tentative plan:
 - ▶ Incentives: 5 weeks.
 - ▶ Information: 5 weeks.
 - ▶ Student presentations: 4 weeks.
 - ▶ Review and preview: 1 week.
- ▶ CEIBA.
 - ▶ Viewing your grades.
- ▶ <http://www.ntu.edu.tw/~lckung/courses/IE16/>.
 - ▶ Downloading course materials.
 - ▶ Linking to lecture videos.
- ▶ <https://piazza.com/ntu.edu.tw/fall2016/im7011/>.
 - ▶ On-line discussions.
 - ▶ Receiving announcements.

Quiz

- ▶ Now it is time for a quiz!

Road map

- ▶ Course overview.
- ▶ **Convexity and optimization.**
- ▶ Applications.

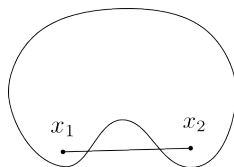
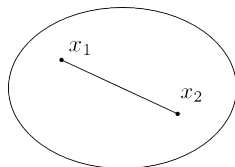
Convex sets

Definition 1 (Convex sets)

A set F is **convex** if

$$\lambda x_1 + (1 - \lambda)x_2 \in F$$

for all $\lambda \in [0, 1]$ and $x_1, x_2 \in F$.



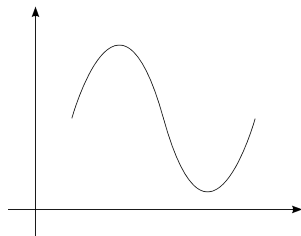
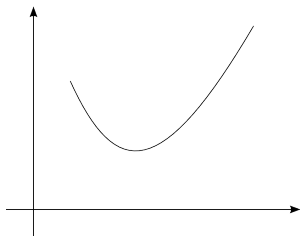
Convex functions

Definition 2 (Convex functions)

For a convex domain F , a function $f(\cdot)$ is **convex** over F if

$$f(\lambda x_1 + (1 - \lambda)x_2) \leq \lambda f(x_1) + (1 - \lambda)f(x_2)$$

for all $\lambda \in [0, 1]$ and $x_1, x_2 \in F$.



Some examples

► Convex sets?

- $X_1 = [10, 20]$.
- $X_2 = (10, 20)$.
- $X_3 = \mathbb{N}$.
- $X_4 = \mathbb{R}$.
- $X_5 = \{(x, y) | x^2 + y^2 \leq 4\}$.
- $X_6 = \{(x, y) | x^2 + y^2 \geq 4\}$.

► Convex functions?

- $f_1(x) = x + 2, x \in \mathbb{R}$.
- $f_2(x) = x^2 + 2, x \in \mathbb{R}$.
- $f_3(x) = \sin(x), x \in (0, 2\pi)$.
- $f_4(x) = \sin(x), x \in (\pi, 2\pi)$.
- $f_5(x) = \log(x), x \in (0, \infty)$.
- $f_6(x, y) = x^2 + y^2, (x, y) \in \mathbb{R}^2$.

Strictly convex and concave functions

Definition 3 (Strictly convex functions)

For a convex domain F , a function $f(\cdot)$ is **strictly convex** over F if

$$f\left(\lambda x_1 + (1 - \lambda)x_2\right) < \lambda f(x_1) + (1 - \lambda)f(x_2)$$

for all $\lambda \in (0, 1)$ and $x_1, x_2 \in F$ such that $x_1 \neq x_2$.

Definition 4 ((Strictly) concave functions)

For a convex domain F , a function $f(\cdot)$ is **(strictly) concave** over F if $-f(\cdot)$ is (strictly) convex.

Derivatives of convex functions

- ▶ In this course, most of the functions are twice-differentiable with continuous second-order derivatives.
- ▶ Recall a function's gradient and Hessian:
 - ▶ For an n -dimensional differentiable function $f(x)$, its **gradient** is the $n \times 1$ vector

$$\nabla f \equiv \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}.$$

- ▶ For an n -dimensional twice-differentiable function $f(x_1, \dots, x_n)$, its **Hessian** is the $n \times n$ matrix

$$\nabla^2 f \equiv \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}.$$

- ▶ (Calculus) If the second-order derivatives are all continuous, the Hessian is symmetric.

Derivatives of convex functions

- ▶ Let f be twice-differentiable with continuous second-order derivatives:

Proposition 1

For $f : \mathbb{R} \rightarrow \mathbb{R}$ over an interval $F \subseteq \mathbb{R}$:

- ▶ f is (strictly) convex over F if and only if $f''(x) \geq 0$ (> 0) for all $x \in F$.
- ▶ f is (strictly) concave over F if and only if $f''(x) \leq 0$ (< 0) for all $x \in F$.

Proposition 2

For $f : \mathbb{R}^n \rightarrow \mathbb{R}$ over a region $F \subseteq \mathbb{R}^n$:

- ▶ f is (strictly) convex over F if and only if $\nabla^2 f(x)$ is positive (semi)definite for all $x \in F$.
 - ▶ f is (strictly) concave over F if and only if $\nabla^2 f(x)$ is negative (semi)definite for all $x \in F$.
- ▶ (Linear Algebra) A symmetric $n \times n$ matrix A is called positive (semi)definite if $y^T A y \geq 0$ (> 0) for all $y \in \mathbb{R}^n$.

Some examples revisited

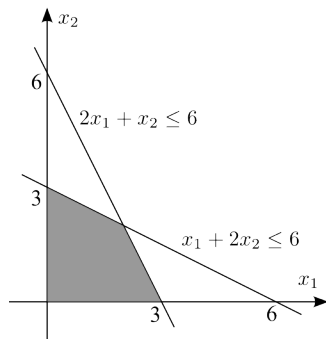
- ▶ $f_1(x) = x + 2, x \in \mathbb{R}: f_1''(x) = 0$, convex and concave.
- ▶ $f_2(x) = x^2 + 2, x \in \mathbb{R}: f_2''(x) = 2 > 0$, strictly convex.
- ▶ $f_3(x) = \sin(x), x \in (0, 2\pi), f_3''(x) = -\sin(x)$, neither.
- ▶ $f_4(x) = \sin(x), x \in (\pi, 2\pi), f_4''(x) = -\sin(x) > 0$, strictly convex.
- ▶ $f_5(x) = \log(x), x \in (0, \infty): f_5''(x) = -\frac{1}{x^2} < 0$, strictly concave.
- ▶ $f_6(x, y) = x^2 + y^2, (x, y) \in \mathbb{R}^2: \nabla^2 f_6(x, y) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ is positive definite, strictly convex.

Linear programming

- ▶ Consider the problem

$$\begin{aligned} z^* = \max \quad & x_1 + x_2 \\ \text{s.t.} \quad & x_1 + 2x_2 \leq 6 \\ & 2x_1 + x_2 \leq 6 \\ & x_i \geq 0 \quad \forall i = 1, 2. \end{aligned}$$

- ▶ The feasible region is the shaded area.
- ▶ An optimal solution is $(x_1^*, x_2^*) = (2, 2)$. Is it unique?
- ▶ The corresponding objective value $z^* = 4$.
- ▶ An optimization problem is a **linear program** (LP) if the objective function and constraints are all linear.



Nonlinear programming

- ▶ An optimization problem is a **nonlinear program** (NLP) if it is not a linear program.
- ▶ Consider the problem

$$\begin{aligned} z^* = \max \quad & x_1 + x_2 \\ \text{s.t.} \quad & x_1^2 + x_2^2 \leq 16 \\ & x_1 + x_2 \geq 1. \end{aligned}$$

- ▶ What is the feasible region?
- ▶ What is an optimal solution? Is it unique?
- ▶ What is the value of z^* ?
- ▶ An optimization problem is a **convex program** if in it we maximize a concave function over a convex feasible region.
- ▶ All convex programs can be solved efficiently.
- ▶ It may not be possible to solve a nonconvex program efficiently.

Infeasible and unbounded problems

- ▶ Not all problems have an optimal solution.
- ▶ A problem is **infeasible** if there is no feasible solution.
 - ▶ E.g., $\max\{x^2 | x \leq 2, x \geq 3\}$.
- ▶ A problem is **unbounded** if given any feasible solution, there is another feasible solution that is better.
 - ▶ E.g., $\max\{e^x | x \geq 3\}$.
 - ▶ How about $\min\{\sin x | x \geq 0\}$?
- ▶ A problem may be infeasible, unbounded, or finitely optimal (i.e., having at least one optimal solution).

Set of optimal solutions

- ▶ The **set of optimal solutions** of a problem $\max\{f(x)|x \in X\}$ is

$$\operatorname{argmax}\{f(x)|x \in X\}.$$

- ▶ For $f(x) = \cos x$ and $X = [0, 2\pi]$, we have

$$\operatorname{argmax}\left\{\cos x \mid x \in [0, 2\pi]\right\} = \{0, 2\pi\}.$$

- ▶ If x^* is an optimal solution of $\max\{f(x)|x \in X\}$, we should write

$$x^* \in \operatorname{argmax}\{f(x)|x \in X\},$$

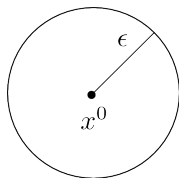
NOT $x^* = \operatorname{argmax}\{f(x)|x \in X\}$!

Global optima

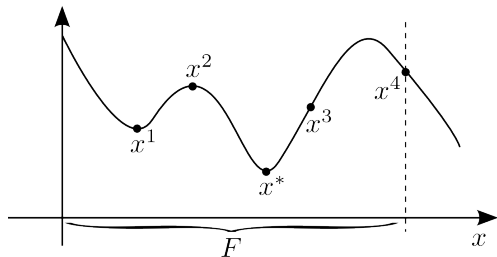
- ▶ For a function $f(x)$ over a feasible region F :
 - ▶ A point x^* is a **global minimum** if $f(x^*) \leq f(x)$ for all $x \in F$.
 - ▶ A point x' is a **local minimum** if for some $\epsilon > 0$ we have

$$f(x') \leq f(x) \quad \forall x \in B(x', \epsilon) \cap F,$$

where $B(x^0, \epsilon) \equiv \{x | d(x, x^0) \leq \epsilon\}$ and $d(x, y) \equiv \sqrt{\sum_{i=1}^n (x_i - y_i)^2}$.



$B(x^0, \epsilon)$



- ▶ **Global maxima** and **local maxima** are defined accordingly.

First-order necessary condition

- ▶ Consider an **unconstrained** problem

$$\max_{x \in \mathbb{R}^n} f(x).$$

Proposition 3 (Unconstrained FONC)

For a differentiable function $f : \mathbb{R}^n \rightarrow \mathbb{R}$, a point x^ is a local maximum of f only if*

- ▶ $f'(x^*) = 0$ if $n = 1$.
- ▶ $\nabla f(x^*) = 0$ if $n \geq 2$.

Examples

- ▶ Consider the problem

$$\max_{x \in \mathbb{R}} x^3 - \frac{9}{2}x^2 + 6x + 2$$

The FONC yields

$$3(x^2 - 3x + 2) = 0.$$

Solving the equation gives us 1 and 2 as two candidates of local maxima.

- ▶ It is easy to see that $x^* = 1$ is a local maxima but $\tilde{x} = 2$ is NOT.

- ▶ Consider the problem

$$\max_{x \in \mathbb{R}^2} f(x) = x_1^2 - x_1x_2 + x_2^2 - 6x_2.$$

The FONC yields

$$\nabla f(x) = \begin{bmatrix} 2x_1 - x_2 \\ -x_1 + 2x_2 - 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Solving the linear system gives us $(2, 4)$ as the only candidate of local maxima.

- ▶ Note that it is NOT necessarily a local maximum!

Second-order necessary condition

- ▶ Let's proceed further.

Proposition 4 (Unconstrained SONC)

Suppose $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is twice-differentiable. For a point x^ to be a local maximum of f , we need:*

- ▶ $f''(x^*) \leq 0$ if $n = 1$.
 - ▶ $\nabla^2 f(x^*)$ is negative semidefinite if $n \geq 2$.
- ▶ Note that we do not need the function to be concave; we only need f'' or $\nabla^2 f$ to be negative or negative definite **at the point** x^* .
 - ▶ In this course, we will not apply the SONC a lot.
 - ▶ Here our point is that a local maximum requires NOT just

$$\frac{\partial^2 f}{\partial x_i^2} \leq 0 \quad \forall i = 1, \dots, n.$$

We want more than candidates!

- ▶ The FONC and SONC produce candidates of local maxima/minima.
- ▶ What's next?
 - ▶ We need some ways to ensure local optimality.
 - ▶ We need to find a global optimal solution.
- ▶ If the function is **convex or concave**, things are much easier:

Proposition 5

For a differentiable convex (concave) function $f : \mathbb{R}^n \rightarrow \mathbb{R}$:

- ▶ x^* is a global minimum (maximum) if and only if $\nabla f(x^*) = 0$.
- ▶ The global optimum is unique if f is strictly convex or concave.

Remarks

- ▶ When you are asked to solve a problem:
 - ▶ First check whether the objective function is convex/concave. If so the problem typically becomes much easier.
- ▶ All the conditions for unconstrained problems apply to **interior** points of a feasible region.
- ▶ One common strategy for solving constrained problems proceeds in the following steps:
 - ▶ **Ignore** all the constraints.
 - ▶ Solve the unconstrained problem.
 - ▶ Verify that the unconstrained optimal solution satisfies all constraints.
- ▶ If the strategy fails, we seek for other ways.

Binding constraints and boundary solutions

- ▶ An optimal solution may lie on the boundary of the feasible region.
 - ▶ It is a **boundary solution** or a **corner solution**.
- ▶ We need to take a look at those **binding** (or **active**) constraints:

Definition 5

Let $g(\cdot) \leq b$ be an inequality constraint and \bar{x} be a solution. $g(\cdot)$ is binding at \bar{x} if $g(\bar{x}) = b$.

- ▶ $x_1 + x_2 \leq 10$ is binding at $(x_1, x_2) = (2, 8)$.
- ▶ $2x_1 + x_2 \geq 6$ is nonbinding at $(x_1, x_2) = (2, 8)$.
- ▶ $x_1 + 3x_2 = 9$ is binding at $(x_1, x_2) = (6, 1)$.
- ▶ Remarks:
 - ▶ An inequality is nonbinding (inactive) at a point if it is strictly satisfied.
 - ▶ An equality constraint is always binding at any feasible solution.

Binding constraints and boundary solutions

- ▶ Consider a **single-dimensional** constrained optimization problem

$$\begin{aligned} \max \quad & f(x) \\ \text{s.t.} \quad & g_i(x) \leq 0 \quad \forall i = 1, \dots, m. \end{aligned}$$

- ▶ If $f(\cdot)$ is strictly concave:
 - ▶ Apply the FOC to obtain a candidate solution \bar{x} .
 - ▶ If \bar{x} is feasible, it is optimal.
 - ▶ Otherwise, the feasible point that is closest to \bar{x} is optimal.
- ▶ In general:
 - ▶ Apply the FONC and SONC to obtain a set of candidate solutions.
 - ▶ Include all the boundary points as candidate solutions.
 - ▶ Compare all the candidate solutions to find an optimal one.
- ▶ For a **multi-dimensional** constrained optimization problem, more advanced techniques are required (e.g., the KKT condition).

Road map

- ▶ Course overview.
- ▶ Convexity and optimization.
- ▶ **Applications.**

Monopoly pricing

- ▶ Suppose a monopolist sells a single product to consumers.
- ▶ Consumers are **heterogeneous** in their **willingness-to-pay**, or **valuation**, of this product.
- ▶ One's valuation, θ , lies on the interval $[0, b]$ uniformly.
 - ▶ He buys the product if and only if his valuation is above the price.
 - ▶ Consumers' decisions are independent.
 - ▶ The total number of consumers is a .
 - ▶ Given a price p , in expectation the number of consumers who buy the product is

$$a \Pr(\theta \geq p) = a \left(1 - \frac{p}{b}\right).$$

- ▶ The unit production cost is c .
- ▶ The seller chooses a unit price p to maximize her total expected profit.

Formulation

- ▶ The **endogenous** decision variable is p .
- ▶ The **exogenous** parameters are a , b , and c .
- ▶ The only constraint is $p \geq 0$.
- ▶ Let $\pi(p)$ be the profit under price p . We have

$$\pi(p) = (p - c)a \left(1 - \frac{p}{b}\right).$$

- ▶ The complete problem formulation is

$$\begin{aligned} \max \quad & (p - c)a \left(1 - \frac{p}{b}\right) \\ \text{s.t.} \quad & p \geq 0. \end{aligned}$$

- ▶ It is without loss of generality to **normalize** the population size a to 1.

Solving the problem

- ▶ Given that $\pi(p) = \frac{a}{b}(p - c)(b - p)$, let's show it is strictly concave:
 - ▶ $\pi'(p) = \frac{a}{b}(b + c - 2p)$.
 - ▶ $\pi''(p) = -2\left(\frac{a}{b}\right) < 0$.
- ▶ Great! Now let's ignore the constraint $p \geq 0$.
- ▶ Applying the FOC, the unconstrained optimal solution is

$$b + c - 2\bar{p} = 0 \quad \Leftrightarrow \quad \bar{p} = \frac{b + c}{2}.$$

- ▶ Does \bar{p} satisfy the ignored constraint? Is it globally optimal?

Managerial/economic implications

- ▶ The optimal price $\bar{p} = \frac{b+c}{2}$ tells us something:
 - ▶ \bar{p} is increasing in the highest possible valuation b . Why?
 - ▶ \bar{p} is increasing in the unit cost c . Why?
 - ▶ \bar{p} has nothing to do with the total number of consumer a . Why?
- ▶ The optimal profit $\pi^* \equiv \pi(\bar{p}) = \frac{a(b-c)^2}{4b}$.
 - ▶ π^* is decreasing in c . Why?
 - ▶ π^* is increasing in a . Why?
 - ▶ How is π^* affected by b ?
 - ▶ Let's answer it:

$$\frac{\partial}{\partial b} \pi^* = \frac{a(b-c)(b+c)}{4b^2} > 0 \quad (\text{why } b > c?).$$

- ▶ It is these **qualitative** managerial/economic implications that matters.
- ▶ Never forget to verify your solutions with your **intuitions**.

Impact of price control

- ▶ Sometimes the price is controlled (e.g., by the government) and has a cap K .
- ▶ For the problem

$$\begin{aligned} \max \quad & (p - c)a \left(1 - \frac{p}{b}\right) \\ \text{s.t.} \quad & p \in [0, K], \end{aligned}$$

the first-order solution $\frac{b+c}{2}$ may be infeasible.

- ▶ The optimal price is

$$p^* = \begin{cases} \frac{b+c}{2} & \text{if } \frac{b+c}{2} \leq K \\ K & \text{otherwise} \end{cases}.$$

Newsvendor problem

- ▶ In some situations, people sell **perishable products**.
 - ▶ They become valueless after the **selling season** is end.
 - ▶ E.g., newspapers become valueless after each day.
 - ▶ High-tech goods become valueless once the next generation is offered.
 - ▶ Fashion goods become valueless when they become out of fashion.
- ▶ In many cases, the seller only have **one chance** for replenishment.
 - ▶ E.g., a small newspaper seller can order only once and obtain those newspapers only at the morning of each day.
- ▶ Often sellers of perishable products face **uncertain demands**.
- ▶ **How many** products one should prepare for the selling season?
 - ▶ Not too many and not too few!

Newsvendor model

- ▶ Let D be the uncertain demand.
- ▶ Let F and f be the cdf and pdf of D (assuming D is continuous).
- ▶ Let r and c be the unit sales revenue and purchasing cost, respectively.
- ▶ Let q be the order quantity.
- ▶ The (expected) profit-maximizing newsvendor solves

$$\max_{q \geq 0} r\mathbb{E}[\min\{q, D\}] - cq.$$

- ▶ Let $\pi(q) = r\mathbb{E}[\min\{q, D\}] - cq$ be the expected profit function.
- ▶ The model can be expanded to include salvage values, disposal fees, shortage costs, etc.

Convexity of the profit function

- ▶ The expected profit function $\pi(q)$ is

$$\begin{aligned}\pi(q) &= r\mathbb{E}[\min\{q, D\}] - cq \\ &= r\left\{\int_0^q xf(x)dx + \int_q^\infty qf(x)dx\right\} - cq \\ &= r\left\{\int_0^q xf(x)dx + q[1 - F(q)]\right\} - cq.\end{aligned}$$

- ▶ We have

$$\pi'(q) = r\left\{qf(q) + 1 - F(q) - qf(q)\right\} - c = r[1 - F(q)] - c$$

and $\pi''(q) = -rf(q) < 0$. $\pi(q)$ is strictly concave.

Optimizing the order quantity

- ▶ Let \bar{q} be the order quantity that satisfies the FOC, we have

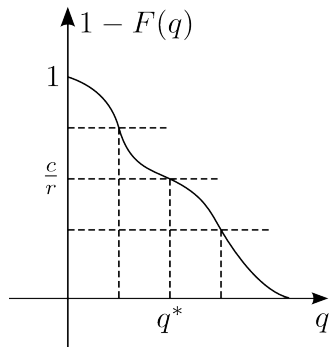
$$r \left[1 - F(\bar{q}) \right] - c = 0$$
$$\Rightarrow F(\bar{q}) = 1 - \frac{c}{r} \quad \text{or} \quad 1 - F(\bar{q}) = \frac{c}{r}.$$

- ▶ Such \bar{q} must be positive (for regular demand distributions).
 - ▶ So \bar{q} is optimal.
 - ▶ The quantity \bar{q} is called the **newsvendor** quantity.
 - ▶ The formula applies to **any** continuous random variable D .

Interpretations of the newsvendor quantity

- ▶ The newsvendor quantity \bar{q} satisfies $1 - F(\bar{q}) = \frac{c}{r}$.
 - ▶ The probability of having a shortage, $1 - F(q)$, is decreasing in q .
- ▶ The optimal quantity \bar{q} is:
 - ▶ Decreasing in c .
 - ▶ Increasing in r .

Does that make economic sense?



Impact of capacity limitation

- ▶ Sometimes the capacity is limited and at most K units can be ordered.
- ▶ For the problem

$$\max_{q \in [0, K]} r \mathbb{E} \left[\min\{q, D\} \right] - cq,$$

the newsvendor quantity \bar{q} satisfying $1 - F(\bar{q}) = \frac{c}{r}$ may be infeasible.

- ▶ The optimal order quantity is

$$q^* = \begin{cases} \bar{q} & \text{if } r[1 - F(K)] - c \leq 0 \\ K & \text{otherwise} \end{cases},$$

where $r[1 - F(K)] - c \leq 0$ is equivalent to $\bar{q} \leq K$ (why?).