Information Economics
Problem Set 3

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1. A manufacturer produces two competing products, A and B, and wants to sell these products to two groups of customers, groups 1 and 2. Group 1 has 1,000 members and group 2 has 1,500 members. The values each customer places on a unit of these two products are shown in the table below. We use V_{ij} to denote a group-i consumer's valuation for product j. For example, $V_{1,A} = \$10$, $V_{2,A} = \$12$, etc.

	Product A	Product B
Group 1	\$10	\$8
Group 2	\$12	\$15

Each consumer will buy at most one unit of either product A or product B. If a group-i consumer buys product j, her consumer surplus is defined to be $V_{ij} - P_j$, where P_j is the price for product j. One will buy the product intended for her if it results in a nonnegative consumer surplus which is no lower than that of buying the other product. Chandler wants to set prices for each product so that group 1 members purchase product A and group 2 members purchase product B. Chandler can only set ONE price for product A and ONE price for product B. It cannot charge different groups different prices for the same product.

   (a) Formulate the manufacturer's pricing problem for maximizing revenues.

   (b) Find the optimal prices analytically.

2. Consider a manufacturer-retailer relationship in a supply chain. The manufacturer produces a product at a unit production cost c and sell it to the retailer. The retailer is a newsvendor facing random market demand $D \sim \text{Uni}(0, 1)$. The unit retail price is fixed to r, which is public to everyone. However, the retailer may be either efficient or inefficient: An efficient retailer sells a product with a unit retail cost d_1 while an inefficient one does so with a unit retail cost d_2, where $d_1 < d_2$. Therefore, the unit "net sales revenues" of an efficient retailer and an inefficient retailer are defined to be $r_H = r - d_1$ and $r_L = r - d_2$. Naturally, $r_L < r_H$. The retailer's retail cost is his private information. The manufacturer believes that the retailer is inefficient with probability β or efficient with probability $1 - \beta$. Each player acts to maximize her/his expected profit. Before the selling season starts, the manufacturer offers the retailer a menu of two contracts, $\{(q_L, t_L), (q_H, t_H)\}$, where (q_i, t_i) is intended for the type-i retailer, $i \in \{L, H\}$. If the retailer selects (q_i, t_i), he obtains q_i units from the manufacturer by paying t_i. He then faces a typical newsvendor situation with q_i units of inventory at the beginning of the selling season.

   (a) In our two-type monopoly pricing model, we assumed that the agent's utility function is $v(q, t, \theta) = \theta v(q) - t$, where θ is the agent's type and $v(\cdot)$ is strictly increasing and strictly concave. For the retailer in this problem, what is θ? What is $v(q)$? Write down the closed-form expression of $v(q)$ as a function of q. Show that it is strictly increasing and strictly concave in the domain of interest.

   (b) By assuming complete information, find the first-best menu for the two types of retailers.

   (c) Suppose there is information asymmetry, formulate the manufacturer's contract design problem whose solution is the second-best menu.

   (d) Continue from Part (c). Suppose $c = 2$, $r_L = 8$, $r_H = 10$, and $\beta = \frac{1}{2}$. Find the manufacturer's second-best menu.

   Note. Plugging in numbers into the formula we derived on slides is fine.

(e) Continue from Part (d). Show that the low-type order quantity is lower than the high-type order quantity. Then show that the low-type order quantity is lower than the efficient level.

3. In Taylor and Xiao (2009), the retailer makes the forecasting decision after the manufacturer makes the offer and before he chooses a contract. Suppose the retailer now makes the forecasting decision before the manufacturer makes the offer. Also suppose that whether the retailer does forecasting is observable. However, the demand signal observed by the retailer is private.

   (a) (5 points) Suppose the retailer does not forecast. Find the manufacturer's optimal offer, the retailer's optimal response, and their equilibrium payoffs. If there are multiple optimal offers or responses, finding an optimal one is enough.

   (b) (10 points) Suppose the retailer forecasts. Find the manufacturer's optimal offer, the retailer's optimal response, and their equilibrium payoffs. If there are multiple optimal offers or responses, finding an optimal one is enough.

   (c) (5 points) Combining your answers in Parts (a) and (b), should the retailer forecast or not? If it depends, find conditions for the retailer to prefer either option.

4. In Pasternack (1985), a contract with full returns with full credits is inefficient. Why in Taylor and Xiao (2009) the full-returns contract is efficient? Find the differences between the two contracts and intuitively explain why one is efficient but the other one is not.

References

Pasternack, B. 1985. Optimal pricing and return policies for perishable commodities. *Marketing Science* **4**(2) 166–176.

Taylor, T., W. Xiao. 2009. Incentives for retailer forecasting: rebates vs. returns. *Management Science* **55**(10) 1654–1669.