Homework Assignment #1

Note

This assignment is due 2PM Thursday, March 5, 2009. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

1. Solve the following exercise problems in Manber's book; you *must use induction* for all proofs:
   2.10 (10 points), 2.18 (10 points), 2.21 (10 points), 2.24 (10 points), 2.30 (Note: *full* binary trees are a more common name for what the problem statement refers to as *complete* binary trees.) (10 points), 2.37 (10 points), 2.39 (20 points).

2. The Harmonic series $H(k)$ is defined by $H(k) = 1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{k-1}+\frac{1}{k}$. Prove *by induction* that $H(2^n) \geq 1 + \frac{n}{2}$, for all $n \geq 0$ (which implies that $H(k)$ diverges). (10 points)

3. Consider the following recurrence relation:

$$\begin{cases} T(0) = 0 \\ T(1) = 1 \\ T(h) = T(h-1) + T(h-2) + 1, \quad h \geq 2 \end{cases}$$

   Prove by induction the relation $T(h) = F(h+2) - 1$, where $F(n)$ is the n-th Fibonacci number (as defined in Chapter 3.5 of Manber's book). (10 points)

4. Bonus problem: 2.23 in Manber's book (again, you must use induction) (20 points)