String Processing

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Data Compression

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Problem

Given a text (a sequence of characters), find an encoding for the characters that satisfies the prefix constraint and that minimizes the total number of bits needed to encode the text.

The *prefix constraint* states that the prefixes of an encoding of one character must not be equal to a complete encoding of another character.

Denote the characters by $c_1, c_2, \cdots, c_n$ and their frequencies by $f_1, f_2, \cdots, f_n$. Given an encoding E in which a bit string s_i represents c_i, the length (number of bits) of the text encoded by using E is $\sum_{i=1}^{n} |s_i| \cdot f_i$.

A Code Tree

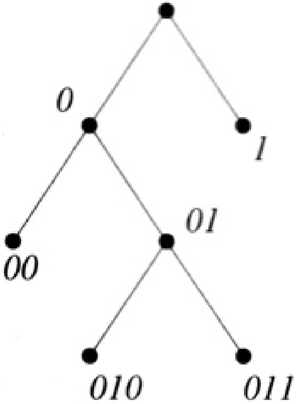

Figure 6.17 The tree representation of encoding.

Source: Manber 1989

A Huffman Tree

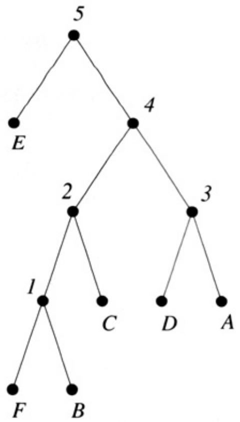

Figure 6.19 The Huffman tree for example 6.1.

Source: Manber 1989

Huffman Encoding

Algorithm Huffman_Encoding (S, f);
  insert all characters into a heap H
    according to their frequencies;
  while H not empty **do**
    if H *contains only one character* X **then**
      make X *the root of* T
    else
      delete X *and* Y *with lowest frequencies*;
        from H;
      create Z *with a frequency equal to the*
        sum of the frequencies of X *and* Y;
      insert Z *into* H;
      make X *and* Y *children of* Z *in* T

String Matching

Problem

Given two strings A $(= a_1 a_2 \cdots a_n)$ and B $(= b_1 b_2 \cdots b_m)$, find the first occurrence (if any) of B in A. In other words, find the smallest k such that, for all i, $1 \leq i \leq m$, we have $a_{k-1+i} = b_i$.

A *substring* of a string A is a consecutive sequence of characters $a_i a_{i+1} \cdots a_j$ from A.

Straightforward String Matching

$A = xyxxyxyxyyxyxyxyyxyxyxx.$ $B = xyxyyxyxyxx.$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
	x	y	x	x	y	x	y	x	y	y	x	y	x	y	x	y	y	x	y	x	y	x	x
1:	x	y	x	y	·	·	·																
2:		x	·	·	·																		
3:			x	y	·	·	·																
4:				x	y	x	y	y	·	·	·												
5:					x	·	·	·															
6:						x	y	x	y	y	x	y	x	y	x	x							
7:							x	·	·	·													
8:								x	y	x	·	·	·										
9:									x	·	·	·											
10:										x	·	·	·										
11:											x	y	x	y	y	·	·	·					
12:												x	·	·	·								
13:													x	y	x	y	y	x	y	x	y	x	x

Figure 6.20 An example of a straightforward string matching.

Source: Manber 1989

Matching Against Itself

$B =$	x	y	x	y	y	x	y	x	y	x	x
		x	·	·	·						
			x	y	x	·	·	·			
				x	·	·	·				
					x	·	·	·			
						x	y	x	y	y	
							x	·	·	·	
								x	y	x	

Figure 6.21 Matching the pattern against itself.

Source: Manber 1989

The Values of *next*

$i =$	1	2	3	4	5	6	7	8	9	10	11
$B =$	x	y	x	y	y	x	y	x	y	x	x
$next =$	-1	0	0	1	2	0	1	2	3	4	3

Figure 6.22 The values of *next*.

Source: Manber 1989

The KMP Algorithm

```
Algorithm String_Match (A, n, B, m);
begin
    j := 1;  i := 1;
    Start := 0;
    while Start = 0 and i ≤ n do
        if B[j] = A[i] then
            j := j + 1;  i := i + 1
        else
            j := next[j] + 1;
            if j = 0 then
                j := 1;  i := i + 1;
        if j = m + 1 then Start := i − m
end
```

The KMP Algorithm (cont.)

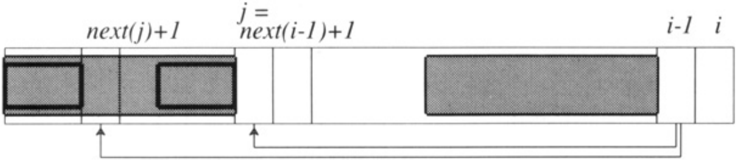

Figure 6.24 Computing next(i).

Source: Manber 1989

The KMP Algorithm (cont.)

```
Algorithm Compute_Next (B, m);
begin
    next[1] := −1;  next[2] := 0;
    for i := 3 to m do
        j := next[i − 1] + 1;
        while b_{i−1} ≠ b_j and j > 0 do
            j := next[j] + 1;
        next[i] := j
end
```

String Editing

Problem

Given two strings A ($= a_1a_2\cdots a_n$) and B ($= b_1b_2\cdots b_m$), find the minimum number of changes required to change A character by character such that it becomes equal to B.

Three types of changes (or edit steps) allowed: (1) insert, (2) delete, and (3) replace.

String Editing (cont.)

Let $C(i,j)$ denote the minimum cost of changing $A(i)$ to $B(j)$, where $A(i) = a_1 a_2 \cdots a_i$ and $B(j) = b_1 b_2 \cdots b_j$.

$$C(i,j) = \min \begin{cases} C(i-1,j)+1 & \text{(deleting } a_i\text{)} \\ C(i,j-1)+1 & \text{(inserting } b_j\text{)} \\ C(i-1,j-1)+1 & (a_i \rightarrow b_j) \\ C(i-1,j-1) & (a_i = b_j) \end{cases}$$

String Editing (cont.)

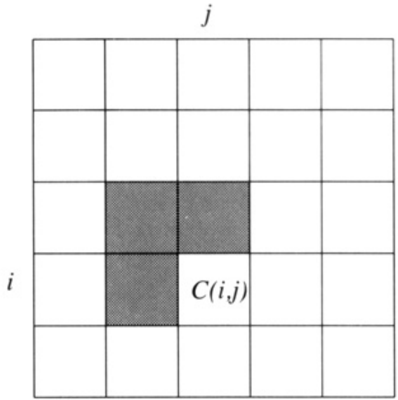

Figure 6.26 The dependencies of $C(i, j)$.

Source: Manber 1989

String Editing (cont.)

Algorithm Minimum_Edit_Distance (A, n, B, m);
    for $i := 0$ **to** n **do** $C[i, 0] := i$;
    for $j := 1$ **to** m **do** $C[0, j] := j$;
    for $i := 1$ **to** n **do**
        for $j := 1$ **to** m **do**
            $x := C[i-1, j] + 1$;
            $y := C[i, j-1] + 1$;
            if $a_i = b_j$ **then**
                $z := C[i-1, j-1]$
            else
                $z := C[i-1, j-1] + 1$;
        $C[i, j] := min(x, y, z)$