Homework Assignment #7

Note

This assignment is due 2:10PM Monday, May 10, 2010. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

There are five problems in this assignment, each accounting for 20 points. There is also a bonus problem, which is worth 20 points.

Problems

1. You are given a set of n coins, among which at least $n-1$ are identical "true" coins and at most one coin is "false", i.e., it is either lighter or heavier than the other true coins. Also, you are given a balance scale, which you may use to compare the total weight of any m coins with that of any other m coins. The n-coins problem is to find the "false" coin, or show that there is no such coin, by making some sequence of comparisons using the balance scale.

   Show that in the worst case it is impossible to solve the n-coins problem with k comparisons if $n > \frac{1}{2}(3^k - 1)$.

2. (6.17) Given two strings *aabccb* and *baacbab*, compute the minimal cost matrix $C[0..6, 0..7]$ for changing the first string character by character to the second one. Aside from giving the cost matrix, please show the details of how the entry $C[6, 7]$ is computed.

3. (6.39) Let A and B be two sets, both with n elements, such that A resides in computer P and B in computer Q. P and Q can communicate by sending messages, and they can perform any kind of local computation. Design an algorithm to find the nth smallest element (i.e., the median) of the union of A and B. You can assume, for simplicity, that all the elements are distinct. Your goal is to minimize the number of messages, where a message can contain one element or one integer. Please present your algorithm in an adequate pseudo code and make assumptions wherever necessary. What is the number of messages in the worst case?

4. (6.40) Design an algorithm that, given a set of integers $S = \{x_1, x_2, \ldots, x_n\}$, finds a nonempty subset $R \subseteq S$, such that

$$\sum_{x_i \in R} x_i \equiv 0 \pmod{n}.$$

5. (6.47) Modify the KMP string matching algorithm to find the largest prefix of B that matches a substring of A. In other words, you do not need to match all of B inside A; instead, you want to find the largest match (but it has to start with b_1).

6. (Bonus) Design an algorithm to arrange a round-robin tournament for any number n (> 1) of players. Each player must play every other player and each play must play one match per round for $n - 1$ rounds. If the number of players is odd, then in each round one player does not participate.