

## Homework Assignment #4

### Note

This assignment is due 2:10PM Monday, March 26, 2012. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

### Problems

There are five problems in this assignment, each accounting for 20 points. (Note: problems marked with "(X.XX)" are taken from [Manber 1989] with probable adaptation.)

1. (5.7) Write a program (or modify the code discussed in class) to recover the solution to a knapsack problem using the *belong* flag. You should make your solution as efficient as possible.
2. (5.8) In algorithm *Knapsack*, we first checked whether the  $i$ th item is unnecessary (by checking  $P[i - 1, j]$ ). If there is a solution with the  $i - 1$  items, we take this solution. We can also make the opposite choice, which is to take the solution with the  $i$ th item if it exists (i.e., check  $P[i, j - k_i]$  first). Which version do you think will have a better performance? Redraw Fig. 5.11 (see slides) to reflect this choice.
3. (5.17) The Knapsack Problem that we discussed in class is defined as follows: Given a set  $S$  of  $n$  items, where the  $i$ th item has an integer size  $S[i]$ , and an integer  $K$ , find a subset of the items whose sizes sum to exactly  $K$  or determine that no such subset exists.

We have described in class an algorithm to solve the problem. Modify the algorithm to solve a variation of the knapsack problem where each item has an *unlimited* supply. In your algorithm, please change the type of  $P[i, k].belong$  into integer and use it to record the number of copies of item  $i$  needed.

4. (5.20) Let  $x_1, x_2, \dots, x_n$  be a set of integers, and let  $S = \sum_{i=1}^n x_i$ . Design an algorithm to partition the set into two subsets of equal sum, or determine that it is impossible to do so. The algorithm should run in time  $O(nS)$ .
5. (5.22) In the **towers of Hanoi** puzzle, there are three pegs  $A$ ,  $B$ , and  $C$ , with  $n$  (generalizing the original eight) disks of different sizes stacked in decreasing order on peg  $A$ . The objective is to transfer all the disks on peg  $A$  to peg  $B$ , moving one disk at a time (from

one peg to one of the other two) and never having a larger disk stacked upon a smaller one.

- (a) Give an algorithm to solve the puzzle. Explain how induction works here.
- (b) Compute the total number of moves in the algorithm. Show the details of your calculation.