

Homework Assignment #8

Note

This assignment is due 2:10PM Monday, May 21, 2012. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

There are five problems in this assignment, each accounting for 20 points. (Note: problems marked with "(X.XX)" are taken from [Manber 1989] with probable adaptation.)

1. (7.1) Consider the problem of finding balance factors in binary trees discussed in class (see slides for "Design by Induction"). Solve this problem using DFS. You need only to define preWORK and postWORK.
2. (7.3) Given as input a connected undirected graph G , a spanning tree T of G , and a vertex v , design an algorithm to determine whether T is a valid DFS tree of G rooted at v . In other words, determine whether T can be the output of DFS under some order of the edges starting with v . The running time of the algorithm should be $O(|V| + |E|)$.
3. (7.6) Consider Algorithm *Single_Source_Shortest_Paths* (discussed in class). Prove that the subgraph consisting of all the edges that belong to shortest paths from v , found during the execution of the algorithm, is a tree rooted at v .
4. (7.28) A **binary de Bruijn sequence** is a (cyclic) sequence of 2^n bits $a_1a_2 \cdots a_{2^n}$ such that each binary string s of size n is represented somewhere in the sequence; that is, there exists a unique index i such that $s = a_i a_{i+1} \cdots a_{i+n-1}$ (where the indices are taken modulo 2^n). For example, the sequence 11010001 is a binary de Bruijn sequence for $n = 3$. Let $G_n = (V, E)$ be a directed graph defined as follows. The vertex set V corresponds to the set of all binary strings of size $n-1$ ($|V| = 2^{n-1}$). A vertex corresponding to the string $a_1a_2 \cdots a_{n-1}$ has an edge leading to a vertex corresponding to the string $b_1b_2 \cdots b_{n-1}$ if and only if $a_2a_3 \cdots a_{n-1} = b_1b_2 \cdots b_{n-2}$. Prove that G_n is a directed Eulerian graph, and discuss the implications for de Bruijn sequences.
5. (7.38) Given a directed acyclic graph $G = (V, E)$, find a simple (directed) path in G that has the maximum number of edges among all simple paths in G . The algorithm should run in linear time.