Homework Assignment #9

Note

This assignment is due 2:10PM Tuesday, May 28, 2013. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

There are five problems in this assignment, each accounting for 20 points. (Note: problems marked with "(X.XX)" are taken from [Manber 1989] with probable adaptation.)

1. (7.9) Prove that if the costs of all edges in a given connected graph are distinct, then the graph has exactly one unique minimum-cost spanning tree.

2. (7.12)

   (a) Give an example of a weighted connected undirected graph $G = (V, E)$ and a vertex v, such that the minimum-cost spanning tree of G is the same as the shortest-path tree rooted at v.

   (b) Give an example of a weighted connected undirected graph $G = (V, E)$ and a vertex v, such that the minimum-cost spanning tree of G is very different from the shortest path tree rooted at v. Can the two trees be completely disjoint?

3. Consider the algorithm discussed in class for determining the strongly connected components of a directed graph. Is the algorithm still correct if we replace the line "$v.high := \max(v.high, w.DFS\_Number)$" by "$v.high := \max(v.high, w.high)$"? Why? Please explain.

4. (7.61) Let $G = (V, E)$ be a connected weighted undirected graph and T be a minimum-cost spanning tree (MCST) of G. Suppose that the cost of one edge $\{u, v\}$ in G is changed (*increased* or *decreased*); $\{u, v\}$ may or may not belong to T. Design an algorithm to either find a new MCST or to determine that T is still an MCST. The more efficient your algorithm is, the more points you will be credited for this problem. Explain why your algorithm is correct and analyze its time complexity.

5. (7.88) Let $G = (V, E)$ be a directed graph, and let T be a DFS tree of G. Prove that the intersection of the edges of T with the edges of any strongly connected component of G form a subtree of T.