# Homework Assignment #1

## Note

This assignment is due 2:10PM Tuesday, March 4, 2014. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

## Problems

There are five problems in this assignment, each accounting for 20 points. You must use *induction* for all proofs. (Note: problems marked with "(X.XX)" are taken from [Manber 1989] with probable adaptation.)

1. (2.11) Find an expression for the sum of the $i$-th row of the following triangle, and prove the correctness of your claim. Each entry in the triangle is the sum of three entries directly above it (a nonexisting entry is considered 0).

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & 1 & 1 & & & \\ & & 1 & 2 & 3 & 2 & 1 & & \\ & 1 & 3 & 6 & 7 & 6 & 3 & 1 & \\ 1 & 4 & 10 & 16 & 19 & 16 & 10 & 4 & 1 \end{array}$$

2. The Harmonic series $H(k)$ is defined by $H(k) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k-1} + \frac{1}{k}$. Prove that $H(2^n) \geq 1 + \frac{n}{2}$, for all $n \geq 0$ (which implies that $H(k)$ diverges).

3. (2.14) Consider the following series: 1, 2, 3, 4, 5, 10, 20, 40, ..., which starts as an arithmetic series, but after the first 5 terms becomes a geometric series. Prove that any positive integer can be written as a sum of distinct numbers from this series.

4. Consider binary trees where every internal node has two children. For any such tree $T$, let $l_T$ denote the number of its leaves and $m_T$ the number of its internal nodes. Prove *by induction* that $l_T = m_T + 1$.

5. (2.37) Consider the recurrence relation for Fibonacci numbers $F(n) = F(n-1) + F(n-2)$. Without solving this recurrence, compare $F(n)$ to $G(n)$ defined by the recurrence $G(n) = G(n-1) + G(n-2) + 1$. It seems obvious that $G(n) > F(n)$ (because of the extra 1). Yet the following is a seemingly valid proof (by induction) that $G(n) = F(n) - 1$. We assume, by induction, that $G(k) = F(k) - 1$ for all $k$ such that $1 \leq k \leq n$, and we consider $G(n+1)$:

$$G(n+1) = G(n) + G(n-1) + 1 = F(n) - 1 + F(n-1) - 1 + 1 = F(n+1) - 1$$

What is wrong with this proof?