Homework Assignment #3

Note

This assignment is due 2:10PM Friday, March 21, 2014. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

There are five problems in this assignment, each accounting for 20 points. (Note: problems marked with "(X.XX)" are taken from [Manber 1989] with probable adaptation.)

1. (3.4) Below is a theorem from Manber's book:

   For all constants $c > 0$ and $a > 1$, and for all monotonically increasing functions $f(n)$, we have $(f(n))^c = O(a^{f(n)})$.

   Prove, by using the above theorem, that for all constants $a, b > 0$, $(\log_2 n)^a = O(n^b)$.

2. (3.5) For each of the following pairs of functions, say whether $f(n) = O(g(n))$ and/or $f(n) = \Omega(g(n))$. Justify your answers.

	$f(n)$	$g(n)$
(a)	$\frac{n}{\log n}$	$(\log n)^2$
(b)	$n^3 2^n$	3^n

3. (3.18) Consider the recurrence relation

   $T(n) = 2T(n/2) + 1$, $T(2) = 1$.

   We try to prove that $T(n) = O(n)$ (we limit our attention to powers of 2). We guess that $T(n) \leq cn$ for some (as yet unknown) c, and substitute cn in the expression. We have to show that $cn \geq 2c(n/2) + 1$. But this is clearly not true. Find the correct solution of this recurrence (you can assume that n is a power of 2), and explain why this attempt failed.

4. Solve the following recurrence relation using *generating functions*. This is a very simple recurrence relation, but you must use generating functions in your solution.

$$\begin{cases} T(1) = 1 \\ T(2) = 3 \\ T(n) = 2T(n-1) - T(n-2), \quad n \geq 3 \end{cases}$$

5. (3.30) Use Equation 1, shown below, to prove that $S(n) = \sum_{i=1}^{n} \lceil \log_2(n/i) \rceil$ satisfies $S(n) = O(n)$.

Bounding a summation by an integral
If $f(x)$ is a monotonically increasing continuous function, then

$$\sum_{i=1}^{n} f(i) \leq \int_{x=1}^{x=n+1} f(x)dx. \tag{1}$$