

Homework Assignment #9

Note

This assignment is due 2:10PM Tuesday, June 3, 2014. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

There are five problems in this assignment, each accounting for 20 points. (Note: problems marked with "(X.XX)" are taken from [Manber 1989] with probable adaptation.)

1. (7.9) Prove that if the costs of all edges in a given connected graph are distinct, then the graph has exactly one unique minimum-cost spanning tree.
2. (7.12)
 - (a) Give an example of a weighted connected undirected graph $G = (V, E)$ and a vertex v , such that the minimum-cost spanning tree of G is the same as the shortest-path tree rooted at v .
 - (b) Give an example of a weighted connected undirected graph $G = (V, E)$ and a vertex v , such that the minimum-cost spanning tree of G is very different from the shortest path tree rooted at v . Can the two trees be completely disjoint?
3. What is wrong with the following algorithm for computing the minimum-cost spanning tree of a given weighted undirected graph (assumed to be connected)?

If the input is just a single-node graph, return the single node. Otherwise, divide the graph into two subgraphs, recursively compute their minimum-cost spanning trees, and then connect the two spanning trees with an edge between the two subgraphs that has the minimum weight.
4. (7.61) Let $G = (V, E)$ be a connected weighted undirected graph and T be a minimum-cost spanning tree (MCST) of G . Suppose that the cost of one edge $\{u, v\}$ in G is changed (*increased* or *decreased*); $\{u, v\}$ may or may not belong to T . Design an algorithm to either find a new MCST or to determine that T is still an MCST. The more efficient your algorithm is, the more points you will be credited for this problem. Explain why your algorithm is correct and analyze its time complexity.
5. Consider the algorithm discussed in class for determining the strongly connected components of a directed graph. Is the algorithm still correct if we replace the line " $v.high := \max(v.high, w.DFS_Number)$ " by " $v.high := \max(v.high, w.high)$ "? Why? Please explain.