

String Processing

(Based on [Manber 1989])

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Data Compression

Problem

Given a text (a sequence of characters), find an encoding for the characters that satisfies the prefix constraint and that minimizes the total number of bits needed to encode the text.

The *prefix constraint* states that the prefixes of an encoding of one character must not be equal to a complete encoding of another character.

Denote the characters by c_1, c_2, \dots, c_n and their frequencies by f_1, f_2, \dots, f_n . Given an encoding E in which a bit string s_i represents c_i , the length (number of bits) of the text encoded by using E is

$$\sum_{i=1}^n |s_i| \cdot f_i.$$

A Code Tree

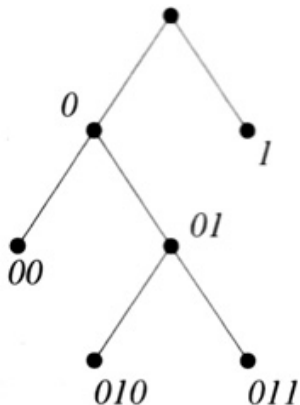


Figure 6.17 The tree representation of encoding.

Source: [Manber 1989].

A Huffman Tree

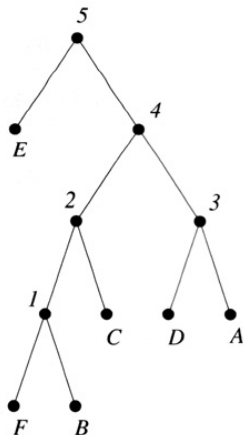


Figure 6.19 The Huffman tree for example 6.1.

Source: [Manber 1989].

Huffman Encoding

Algorithm Huffman_Encoding (S, f);

*insert all characters into a heap H
according to their frequencies;*

while H not empty **do**

if H contains only one character X **then**
make X the root of T

else

*delete X and Y with lowest frequencies;
from H ;*

*create Z with a frequency equal to the
sum of the frequencies of X and Y ;*

insert Z into H ;

make X and Y children of Z in T

String Matching

Problem

Given two strings $A (= a_1a_2 \cdots a_n)$ and $B (= b_1b_2 \cdots b_m)$, find the first occurrence (if any) of B in A . In other words, find the smallest k such that, for all i , $1 \leq i \leq m$, we have $a_{k-1+i} = b_i$.

A *substring* of a string A is a consecutive sequence of characters $a_i a_{i+1} \cdots a_j$ from A .

Straightforward String Matching

$A = xyxxxyxyxyxyxyxyxyxx$. $B = xyxyxyxyxx$.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	
	x	y	x	x	y	x	y	x	y	y	x	y	x	y	x	y	y	x	y	x	y	x	x	
1:	x	y	x	y	.	.	.																	
2:		x	.	.	.																			
3:			x	y	.	.	.																	
4:				x	y	x	y	y												
5:					x															
6:						x	y	x	y	y	x	y	x	y	x	x								
7:							x													
8:								x	y	x										
9:									x											
10:										x										
11:											x	y	x	y	y					
12:												x								
13:														x	y	x	y	y	x	y	x	y	x	x

Figure 6.20 An example of a straightforward string matching.

Matching Against Itself

$$\begin{array}{cccccccccccc}
 B = & x & y & x & y & y & x & y & x & y & x & x \\
 & & x & \cdot & \cdot & \cdot & & & & & & \\
 & & & x & y & x & \cdot & \cdot & \cdot & & & \\
 & & & & x & \cdot & \cdot & \cdot & & & & \\
 & & & & & x & \cdot & \cdot & \cdot & & & \\
 & & & & & & x & y & x & y & y & \\
 & & & & & & & x & \cdot & \cdot & \cdot & \\
 & & & & & & & & x & y & x &
 \end{array}$$

Figure 6.21 Matching the pattern against itself.

Source: [Manber 1989].

The Values of *next*

<i>i</i> =	1	2	3	4	5	6	7	8	9	10	11
<i>B</i> =	<u>x</u>	y	x	y	y	x	y	x	y	x	x
<i>next</i> =	-1	0	0	1	2	0	1	2	3	4	3

Figure 6.22 The values of *next*.

Source: [Manber 1989].

The KMP Algorithm

```
Algorithm String_Match ( $A, n, B, m$ );  
begin  
   $j := 1; i := 1;$   
   $Start := 0;$   
  while  $Start = 0$  and  $i \leq n$  do  
    if  $B[j] = A[i]$  then  
       $j := j + 1; i := i + 1$   
    else  
       $j := next[j] + 1;$   
      if  $j = 0$  then  
         $j := 1; i := i + 1;$   
      if  $j = m + 1$  then  $Start := i - m$   
end
```

The KMP Algorithm (cont.)

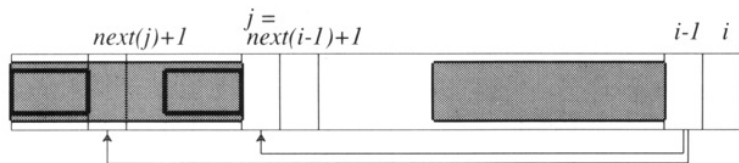


Figure 6.24 Computing $next(i)$.

Source: [Manber 1989].

The KMP Algorithm (cont.)

```
Algorithm Compute_Next ( $B, m$ );  
begin  
   $next[1] := -1$ ;  $next[2] := 0$ ;  
  for  $i := 3$  to  $m$  do  
     $j := next[i - 1] + 1$ ;  
    while  $B[i - 1] \neq B[j]$  and  $j > 0$  do  
       $j := next[j] + 1$ ;  
     $next[i] := j$   
end
```

Problem

Given two strings $A (= a_1a_2 \cdots a_n)$ and $B (= b_1b_2 \cdots b_m)$, find the minimum number of changes required to change A character by character such that it becomes equal to B .

Three types of changes (or edit steps) allowed: (1) **insert**, (2) **delete**, and (3) **replace**.

String Editing (cont.)

Let $C(i, j)$ denote the minimum cost of changing $A(i)$ to $B(j)$, where $A(i) = a_1 a_2 \cdots a_i$ and $B(j) = b_1 b_2 \cdots b_j$.

$$C(i, j) = \min \begin{cases} C(i-1, j) + 1 & (\text{deleting } a_i) \\ C(i, j-1) + 1 & (\text{inserting } b_j) \\ C(i-1, j-1) + 1 & (a_i \rightarrow b_j) \\ C(i-1, j-1) & (a_i = b_j) \end{cases}$$

String Editing (cont.)

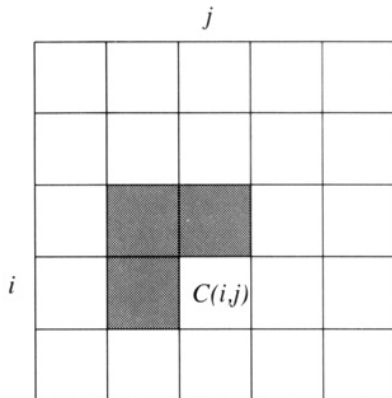


Figure 6.26 The dependencies of $C(i, j)$.

Source: [Manber 1989].

String Editing (cont.)

Algorithm Minimum_Edit_Distance (A, n, B, m);

```
for  $i := 0$  to  $n$  do  $C[i, 0] := i$ ;  
for  $j := 1$  to  $m$  do  $C[0, j] := j$ ;  
for  $i := 1$  to  $n$  do  
  for  $j := 1$  to  $m$  do  
     $x := C[i - 1, j] + 1$ ;  
     $y := C[i, j - 1] + 1$ ;  
    if  $a_i = b_j$  then  
       $z := C[i - 1, j - 1]$   
    else  
       $z := C[i - 1, j - 1] + 1$ ;  
     $C[i, j] := \min(x, y, z)$ 
```