Homework Assignment #9

Note

This assignment is due 2:10PM Tuesday, June 2, 2015. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

There are five problems in this assignment, each accounting for 20 points. (Note: problems marked with "(X.XX)" are taken from [Manber 1989] with probable adaptation.)

1. (7.9) Prove that if the costs of all edges in a given connected graph are distinct, then the graph has exactly one unique minimum-cost spanning tree.

2. (7.12)

   (a) Give an example of a weighted connected undirected graph $G = (V, E)$ and a vertex v, such that the minimum-cost spanning tree of G is the same as the shortest-path tree rooted at v.

   (b) Give an example of a weighted connected undirected graph $G = (V, E)$ and a vertex v, such that the minimum-cost spanning tree of G is very different from the shortest path tree rooted at v. Can the two trees be completely disjoint?

3. (7.16 modified)

   (a) Run the strongly connected components algorithm on the directed graph shown in Figure 1. When traversing the graph, the algorithm should follow the given DFS numbers. Show the *High* values as computed by the algorithm in each step.

   (b) Add the edge $(4, 2)$ to the graph and discuss the changes this makes to the execution of the algorithm.

4. What is wrong with the following algorithm for computing the minimum-cost spanning tree of a given weighted undirected graph (assumed to be connected)?

   If the input is just a single-node graph, return the single node. Otherwise, divide the graph into two subgraphs, recursively compute their minimum-cost spanning trees, and then connect the two spanning trees with an edge between the two subgraphs that has the minimum weight.

5. (7.88) Let $G = (V, E)$ be a directed graph, and let T be a DFS tree of G. Prove that the intersection of the edges of T with the edges of any strongly connected component of G form a subtree of T.

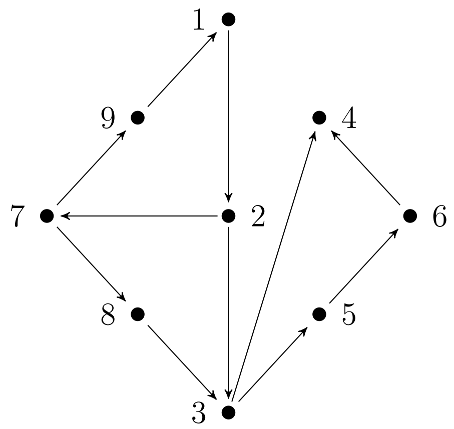

Figure 1: A directed graph with DFS numbers