Homework Assignment #3

Due Time/Date

2:20PM Tuesday, September 27, 2022. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file. Name your file according to this pattern: "b107050xx-hw3". Upload the PDF file to the NTU COOL site for Algorithms 2022. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

There are five problems in this assignment, each accounting for 20 points. (Note: problems marked with "(X.XX)" are taken from [Manber 1989] with probable adaptation.)

1. (3.4) Below is a theorem from Manber's book:

   > For all constants $c > 0$ and $a > 1$, and for all monotonically increasing functions $f(n)$, we have $(f(n))^c = o(a^{f(n)})$.

   Prove, by using the above theorem, that for all constants $a, b > 0$, $(\log_2 n)^a = o(n^b)$.

2. (3.5) For each of the following pairs of functions, say whether $f(n) = O(g(n))$ and/or $f(n) = \Omega(g(n))$. Justify your answers.

	$f(n)$	$g(n)$
(a)	$\frac{n^2}{\log n}$	$n(\log n)^2$
(b)	$n^3 2^n$	3^n

3. Suppose f is a strictly increasing function that maps every positive integer to another positive integer, i.e., if $1 \leq n_1 < n_2$, then $1 \leq f(n_1) < f(n_2)$, and $f(n) = O(g(n))$ for some other function g. Is it true that $\log f(n) = O(\log g(n))$? Please justify your answer. How about $2^{f(n)} = O(2^{g(n)})$? Is it true?

4. (3.18) Consider the recurrence relation

   $$T(n) = 2\ T(n/2) + 1,\ T(2) = 1.$$

   We try to prove that $T(n) = O(n)$ (we limit our attention to powers of 2). We guess that $T(n) \leq cn$ for some (as yet unknown) c, and substitute cn in the expression. We have to show that $cn \geq 2c(n/2) + 1$. But this is clearly not true. Find the correct solution of this recurrence (you can assume that n is a power of 2), and explain why this attempt failed.

5. Solve the following recurrence relation using *generating functions*. This is a very simple recurrence relation, but for the purpose of practicing you must use generating functions in your solution.

$$\begin{cases} T(1) = 1 \\ T(2) = 3 \\ T(n) = T(n-1) + 2\ T(n-2), \quad n \geq 3 \end{cases}$$