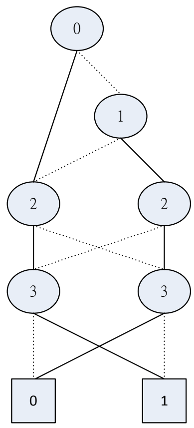
Final

Note

This is an open-book exam. You may consult any books, papers, or notes, but discussion with others is strictly forbidden.

Problems

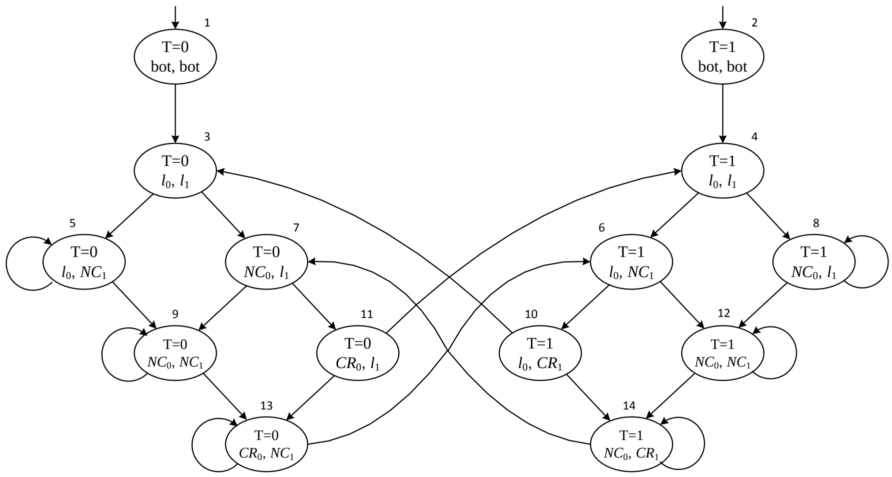
1. Consider the following binary decision diagram (BDD) where a false branch is represented by a dotted line and a true branch by a solid line.



   (a) (10 %) Recover in a systematic way the boolean function represented by the BDD; use x_0, x_1, x_2, and x_3 to name the boolean variables.

   (b) (10 %) Draw a BDD (in canonical form) for the same function but with a different variable ordering: 2, 3, 0, 1.

2. Consider a system with two processes (0 and 1) that repeatedly attempt to enter the critical section via the arbitration of a boolean variable T. Process 0 may enter the critical section when $T = 0$ and it changes T to 1 when exiting the critical section; analogous for Process 1. This system may be modeled as the following Kripke structure.



(a) (15 %) Check if the system satisfies the CTL formula $\mathbf{AG}((pc_0 = NC_0) \rightarrow \mathbf{EF}(pc_0 = CR_0))$ (using the procedures in [CGP; Chapter 4.1]).

(b) (15 %) Use the symbolic CTL model checking algorithm in [CGP; Chapter 6] to compute the states that satisfy the CTL formula $\mathbf{AG}((pc_0 = NC_0) \rightarrow \mathbf{AF}(pc_0 = CR_0))$.

3. We are given an arbitrary Büchi automaton $B = (\Sigma, Q, \Delta, q_0, F)$, where $\Delta \subseteq Q \times \Sigma \times Q$ and $F \subseteq Q$. Define a binary (or transition) relation *pre* on Q such that $(q, q') \in pre$ iff $(q, a, q') \in \Delta$ for some $a \in \Sigma$, so that, in the notation of μ-calculus, $\langle pre \rangle Q'$ will represent the set of automaton states that may reach Q' in one step (after consuming an input symbol). Let *post* be the inverse of *pre*.

   (a) (5 %) Find a suitable μ-calculus expression for the set of states from which some set of states $Q' \subseteq Q$ can be reached (by following the *pre* relation and consuming some input word).

   (b) (10 %) Find a suitable μ-calculus expression for the set of states from which some nontrivial strongly connected component containing a state in F can be reached.

4. (5 %) Are the following two temporal properties the same? If not, what is the difference?

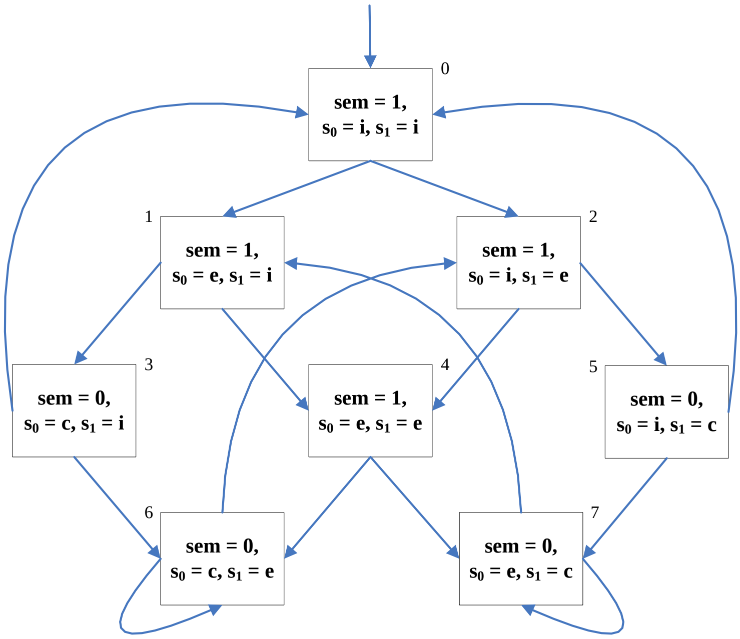
   (a) p holds in every even position of a computation, where the positions are counted from 0.

   (b) $p \wedge \mathbf{G}(p \rightarrow \mathbf{XX}p)$ (or $p \wedge \Box(p \rightarrow \bigcirc\bigcirc p)$)

5. (10 %) It is possible to check the emptiness of a generalized Büchi automaton, without first converting it to a Büchi automaton. Please design an algorithm as efficient as possible for this purpose.

6. (20 %) Solve two of the following three problems:

(a) Consider a system with two processes (0 and 1) that repeatedly attempt to enter the critical section via the arbitration of a binary semaphore. This system may be modeled as the following Kripke structure.



Define a system in the modeling language of SMV (or NuSMV) so that it behaves as in the above Kripke structure. Please try to be as succinct as possible.

(b) Do the same as above, but in the modeling language of SPIN (i.e., Promela). Please try to be as succinct as possible.

(c) Illustrate the DPLL algorithm by checking the satisfiability of $(a \vee \overline{b} \vee c) \wedge (\overline{a} \vee b \vee \overline{c}) \wedge (\overline{a} \vee c \vee \overline{d}) \wedge (\overline{b} \vee \overline{c} \vee d) \wedge (\overline{c} \vee \overline{d})$.