Compositional Reasoning

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Compositional Verification

- **Verification Task:** verify if the system composed of components M_1 and M_2 satisfies a property P, i.e., $M_1 || M_2 \models P$.
- M_1 and M_2 may rely on each other to satisfy P.
- So, it is usually not possible to verify M_1 and M_2 separately.

Component M_1		Component M_2		
Out x : Boolean; In y : Boolean; Init x = true; Repeat forever x:=y;	$\|\|$	Out y : Boolean; Init y = true; Repeat forever y := true;	$\models$	always x=true & y=true

M_1 alone does not guarantee "always x = true"!

Compositional Verification (cont.)

- **Assume-Guarantee** reasoning:

$$\frac{M_1 \parallel A \models P \qquad M_2 \models A}{M_1 \parallel M_2 \models P}$$

- Here, we assume M_1, M_2, A, and P are finite automata.
- If a small A (an abstraction of M_2) exists, then the overall verification task may become easier.

But, how to find A automatically?

A Language-Theoretic Framework

$$\frac{M1 \parallel A \models P \qquad M2 \models A}{M1 \parallel M2 \models P}$$

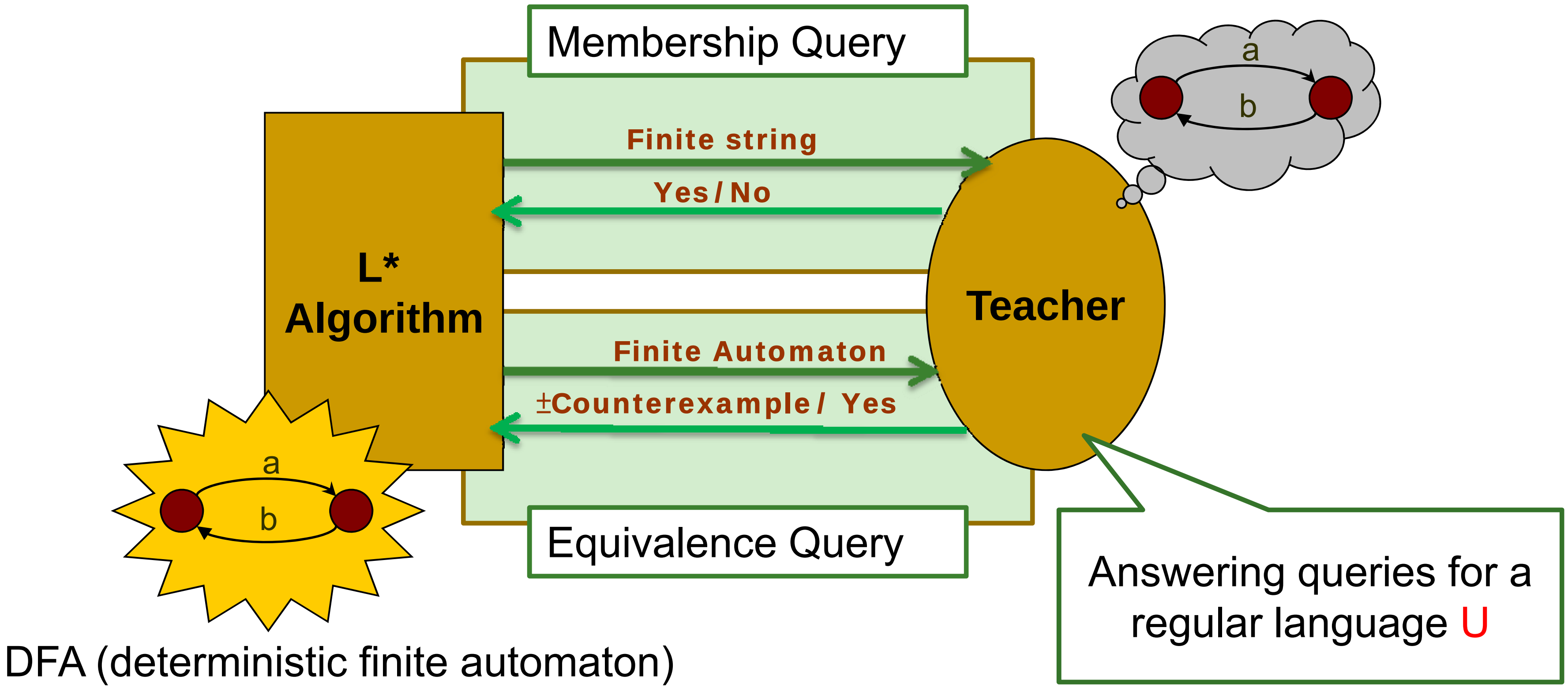
- The **behaviors of components** and **properties** are described as **regular languages.**
- **Parallel composition** is presented by **the intersection of the languages.**
- **A system satisfies a property** if **the language of the system is a subset of the language of the property.**

$$\frac{M1 \cap A \subseteq P \qquad M2 \subseteq A}{M1 \cap M2 \subseteq P}$$

Outline

- Learning-Based Compositional Model Checking:
  - Automation by Learning
  - The L* Algorithm
  - The Problem of L*-Based Approaches
- Learning Minimal Separating DFA's:
  - The L^{SEP} Algorithm
  - Comparison with Another Algorithm
  - Adapt L^{SEP} for Compositional Model Checking

Overview of the L* Algorithm

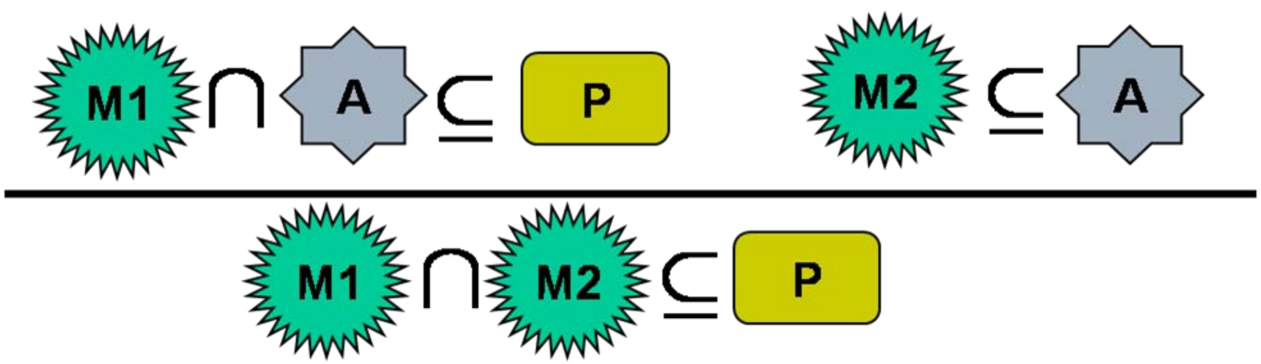


If such a teacher is provided, L* guarantees to produce a DFA that recognizes U using a polynomial number of queries.

Automation by Learning

- **First developed** by Cobleigh, Giannakopoulou, and Pasareanu [TACAS 2003]

- Apply the L^* learning algorithm for regular languages to find an A for the assume-guarantee rule:



The Algorithm of Cobleigh *et al.*

- Automatically find an A for the following assume-guarantee rule:

$M1 \cap A \subseteq P \Leftrightarrow$

$M1 \cap A \cap \overline{P} = \emptyset \Leftrightarrow$

$A \cap \overline{(P \cup \overline{M1})} = \emptyset \Leftrightarrow$

$A \subseteq P \cup \overline{M1}$

$$\frac{M1 \cap A \subseteq P \qquad M2 \subseteq A}{M1 \cap M2 \subseteq P}$$

When $A = P \cup \overline{M1}$,

$M2 \subseteq A \Leftrightarrow$

$M2 \subseteq P \cup \overline{M1} \Leftrightarrow$

$M2 \cap \overline{(P \cup \overline{M1})} = \emptyset \Leftrightarrow$

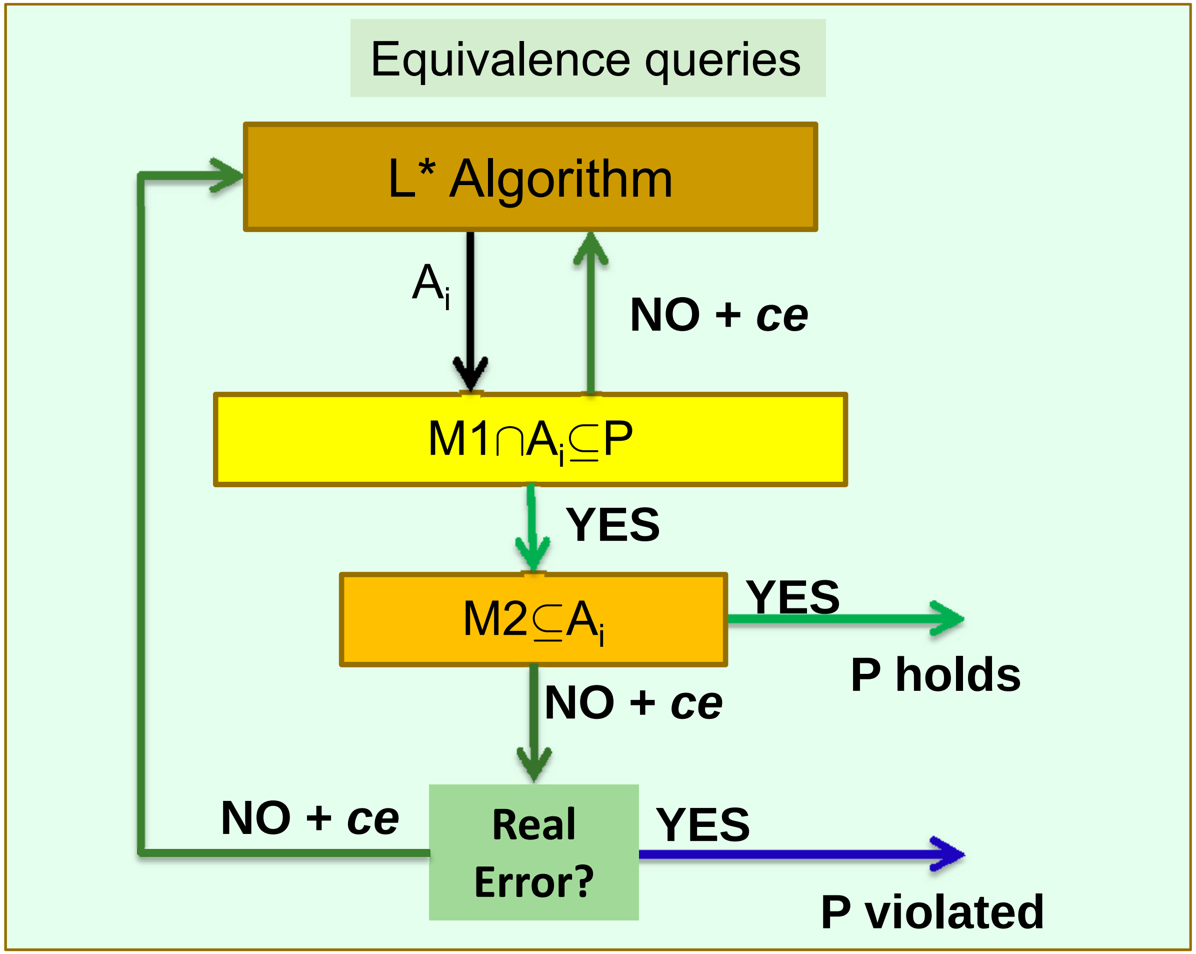
$M2 \cap \overline{P} \cap M1 = \emptyset \Leftrightarrow$

$M1 \cap M2 \subseteq P$

- Apply L* to find it iteratively.
- The target language is $\mathbf{P \cup \overline{M1}}$, the *weakest assumption* for the premise $\mathbf{M1 \cap A \subseteq P}$.

The Algorithm of Cobleigh *et al.* (cont.)

Target: $P \cup \overline{M1}$

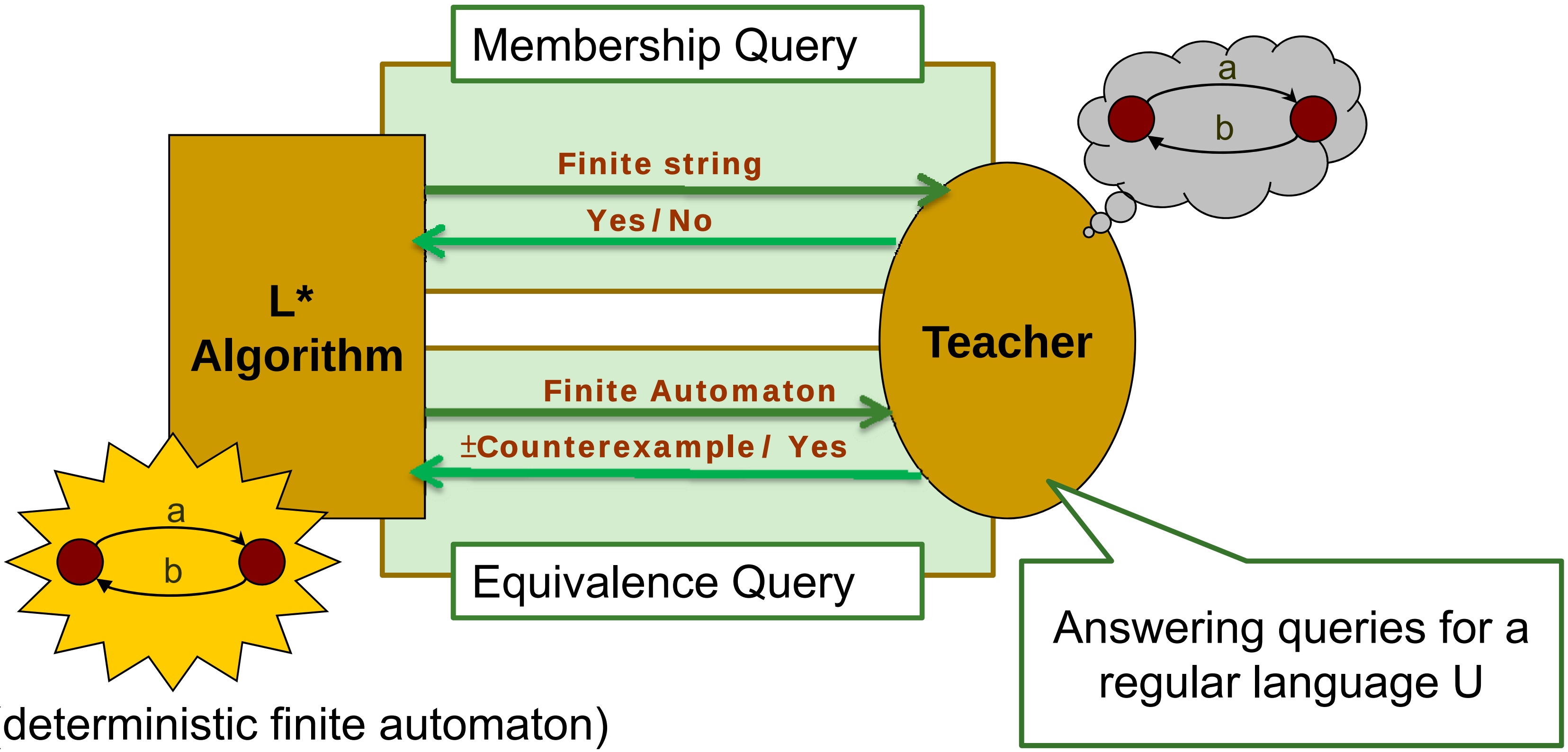


[Cobleigh, Giannakopoulou, and Pasareanu 2003]

Note: *ce* is a real error if *ce* is in M2, but not in $P \cup \overline{M1}$.

The L* Learning Algorithm

- Proposed by D. Angluin [Info.&Comp. 1987] and improved by Rivest and Schapire [Info.&Comp. 1993]



DFA (deterministic finite automaton)

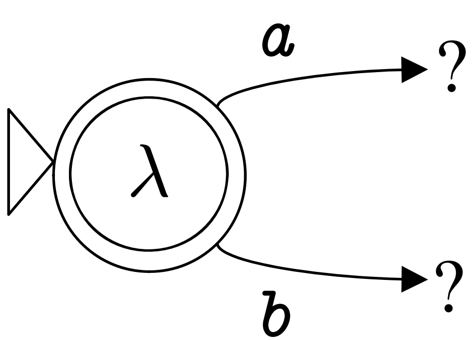
L*: Initial Setting

E: Distinguishing Experiments

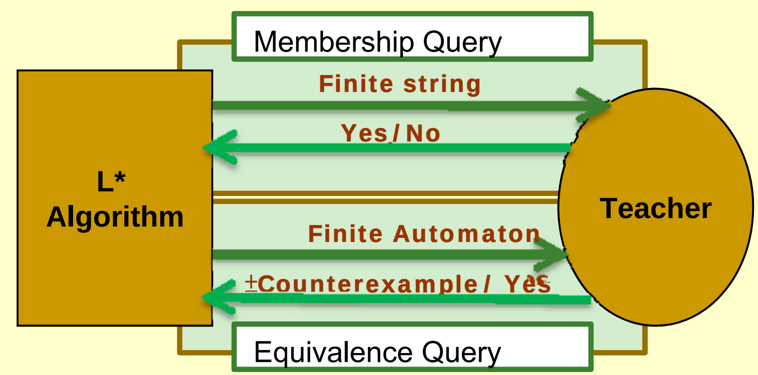
	λ
λ	
a	
b	

States S: λ

Next States $S\Sigma$: a, b



A reminder:



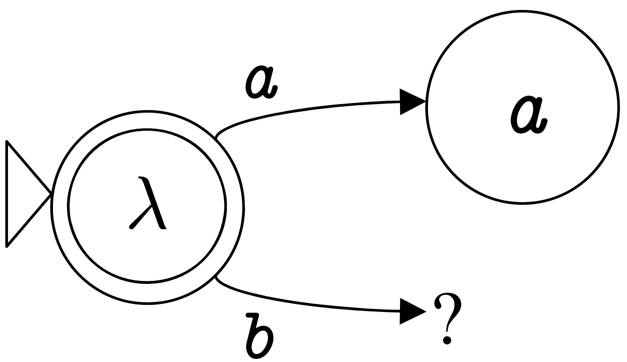
Target: $(ab+aab)^*$

L*: Fill Up the Table by Membership Queries

	λ
λ	T
a	F
b	F

Fill up the table using **membership queries**

a represents a new equivalence class, because its **row** is different from all of those in the current S set.



Target: $(ab+aab)^*$

L*: Table Expansion

Move a to the S set and expand the table with elements aa and ab

	λ
λ	T
a	F
b	F
aa	
ab	

Target: $(ab+aab)^*$

L*: A Closed Table

	λ
λ	T
a	F
b	F
aa	F
ab	T

Again, fill up the table using membership queries

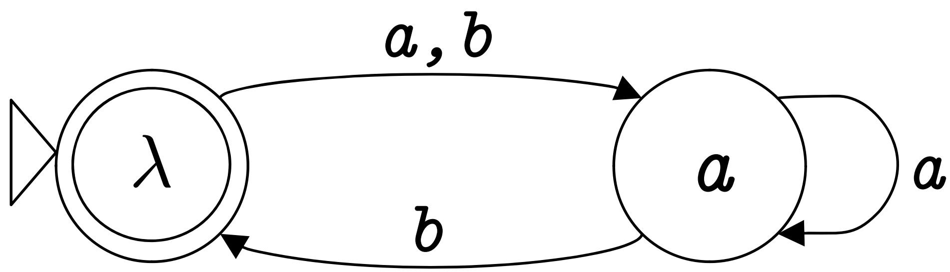
We say that the table is **closed** because every row in the SΣ set appears somewhere in the S set

Target: $(ab+aab)^*$

L*: Making a Conjecture

	λ
λ	T
a	F
b	F
aa	F
ab	T

Construct a DFA from the learned equivalence classes



Counterexample: bb

$\delta(s, a) = s'$ iff sa and s' have the same row.

A suffix b is extracted from bb as a valid distinguishing experiment

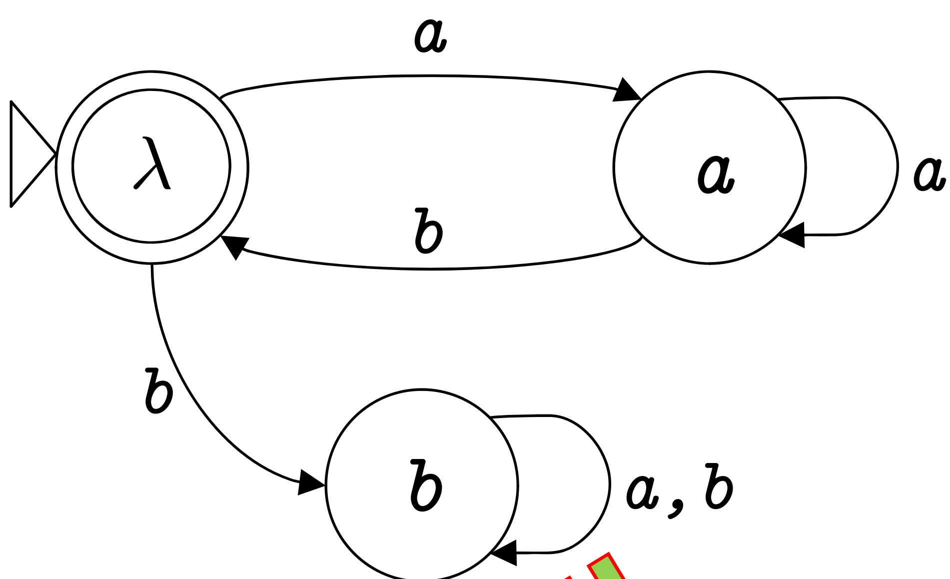
Target: $(ab + aab)^*$

Theorem:
At least one suffix of the counterexample is a valid distinguishing experiment

L*: 2nd Iteration

Add b to the E set, fill up and expand the table following the same procedure

	λ	b
λ	T	F
a	F	T
b	F	F
aa	F	T
ab	T	F
ba	F	F
bb	F	F



Counterexample: $aaab$

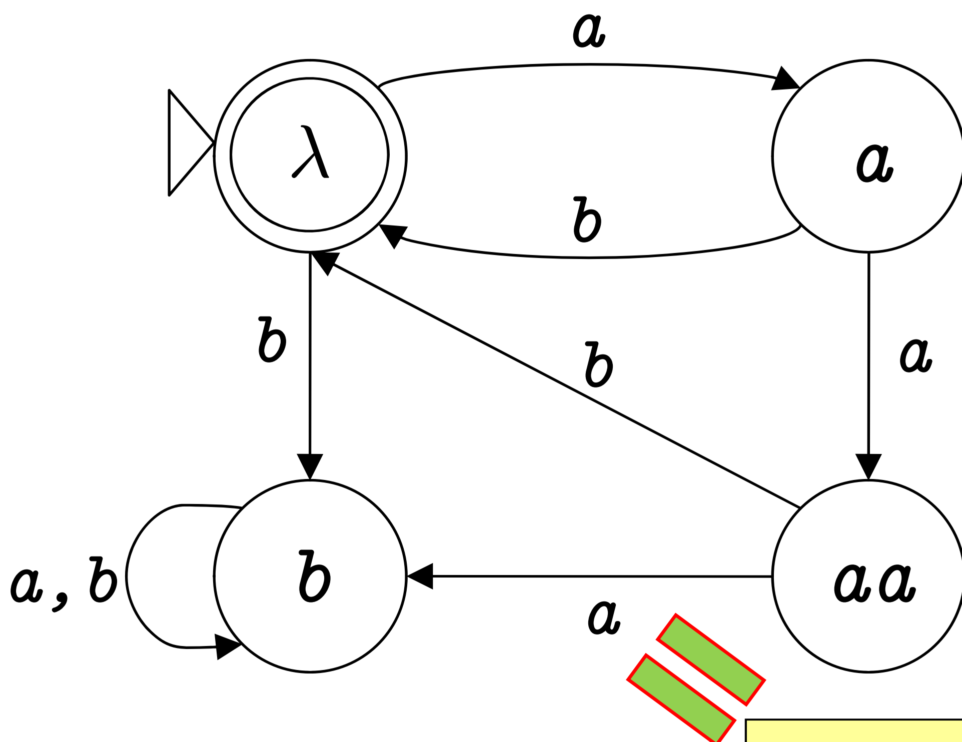
A suffix ab is extracted from $aaab$ as a valid distinguishing experiment

Target: $(ab+aab)^*$

L*: 3rd Iteration (Completed)

Add ab to the E set, fill up and expand the table following the same procedure

	λ	b	ab
λ	T	F	T
a	F	T	T
b	F	F	F
aa	F	T	F
ab	T	F	T
ba	F	F	F
bb	F	F	F
aaa	F	F	F
aab	T	F	T



Target: $(ab+aab)^*$

Theorem:
The DFA produced by L* is the minimal DFA that recognizes that target language

L*: Complexity

- **Complexity:**
  - Equivalence query: at most n-1
  - Membership query: $O(|\Sigma|n^2 + n \log m)$

	λ	b	ab
λ	T	F	T
a	F	T	T
b	F	F	F
aa	F	T	F
ab	T	F	T
ba	F	F	F
bb	F	F	F
aaa	F	F	F
aab	T	F	T

Note: ***n*** is the size of the minimal DFA that recognizes U, ***m*** is the length of the longest counterexample returned from the teacher.

The Problem

- The L*-based approaches cannot guarantee finding the **minimal assumption** (in size), even if there exists one.

$$\frac{M1 \cap A \subseteq P \qquad M2 \subseteq A}{M1 \cap M2 \subseteq P}$$

- The smaller the size of A is, the easier it is to check the correctness of the two premises.

- L^* targets a single language, however, there exists a range of languages that satisfy the premises of an A-G rule.

Finding a Minimal Assumption

- **A reminder**: we use the following Assume-Guarantee rule for decomposition.

$M1 \cap A \subseteq P \Leftrightarrow$

$M1 \cap A \cap \overline{P} = \emptyset \Leftrightarrow$

$A \cap \overline{(P \cup \overline{M1})} = \emptyset \Leftrightarrow$

$A \subseteq P \cup \overline{M1}$

$$\frac{M1 \cap A \subseteq P \qquad M2 \subseteq A}{M1 \cap M2 \subseteq P}$$

- The two premises can be rewritten as follows:

$$M2 \subseteq A \subseteq P \cup \overline{M1}$$

Finding a Minimal Assumption (cont.)

- The two premises can be rewritten as follows:

$$\mathrm{M2} \subseteq \mathrm{A} \subseteq \mathrm{P} \cup \overline{\mathrm{M1}}$$

- **The verification problem** reduces to **finding a minimal separating DFA** that
  - **accepts** every string in **M2** and
  - **rejects** every string not in **P** $\cup$ $\overline{\textbf{M1}}$.

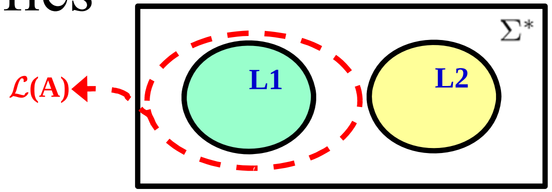
First observed by Gupta, McMillan, and Fu

Learning a Minimal Separating DFA

- **Our contribution:** a **polynomial-query** learning algorithm, L^{Sep}, for minimal separating DFA's.

- **Problem:** given two disjoint regular languages **L1** and **L2**, we want to find a minimal DFA **A** that satisfies

$$\mathbf{L1} \subseteq \mathcal{L}(\mathbf{A}) \subseteq \overline{\mathbf{L2}}$$

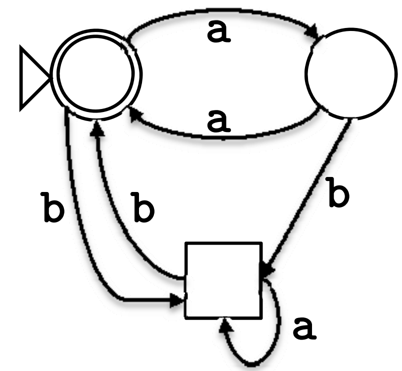


We say that **A** is a separating DFA for L1 and L2

- **Assumption:** a teacher for L1 and L2:
  - Membership query: if a string **s** is in L1 (resp. L2)
  - Containment query: ?⊆L1 , ?⊇L1, ?⊆L2, and ?⊇L2

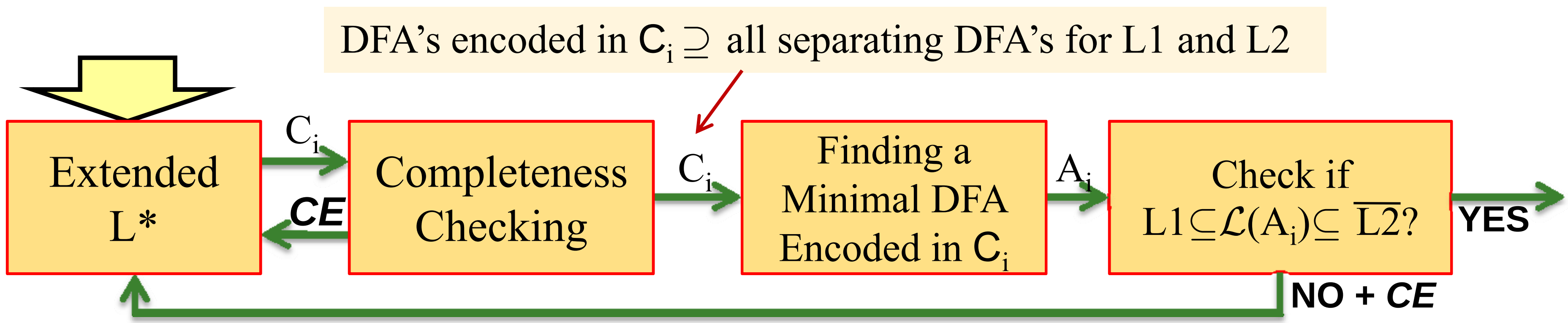
3-Value DFA (3DFA)

- A 3DFA is a tuple $\mathcal{C} = (\Sigma, S, s_0, \delta, Acc, Rej, Dont)$.
- A **DFA** ***A*** is **encoded in** a **3DFA** ***C*** iff ***A***
  - accepts all strings that ***C*** accepts and
  - rejects all strings that ***C*** rejects.
  - A don't care string in ***C*** can be either accepted or rejected by ***A***.

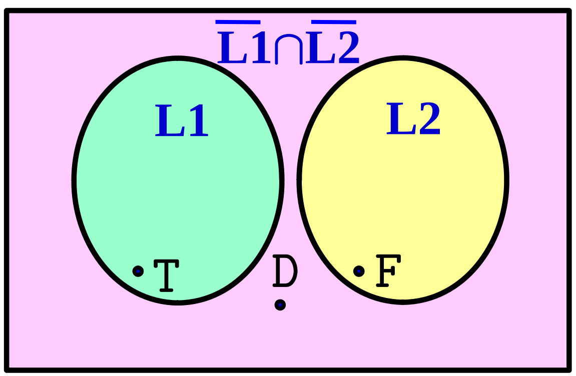


An example of a 3DFA

The L^{Sep} Algorithm: Overview



Target:



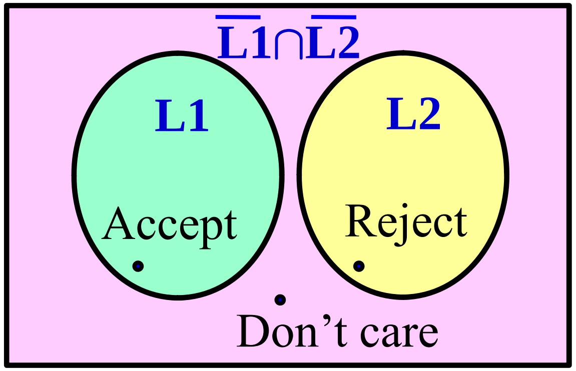
	λ	a
λ	T	F
a	F	T
ab	D	D
b	D	D
aa	T	F
aba	D	D
abb	T	F

Extend the the **L* algorithm** to allow **don't care** values

The Target 3DFA

- The target 3DFA ***C***
  - **accepts** every string in **L1**, and
  - **rejects** every string in **L2**.
  - Strings in $\overline{\mathbf{L1}}\cap\overline{\mathbf{L2}}$ are **don't care** strings.

DFA's encoded in C =
all separating DFA's for L1 and L2

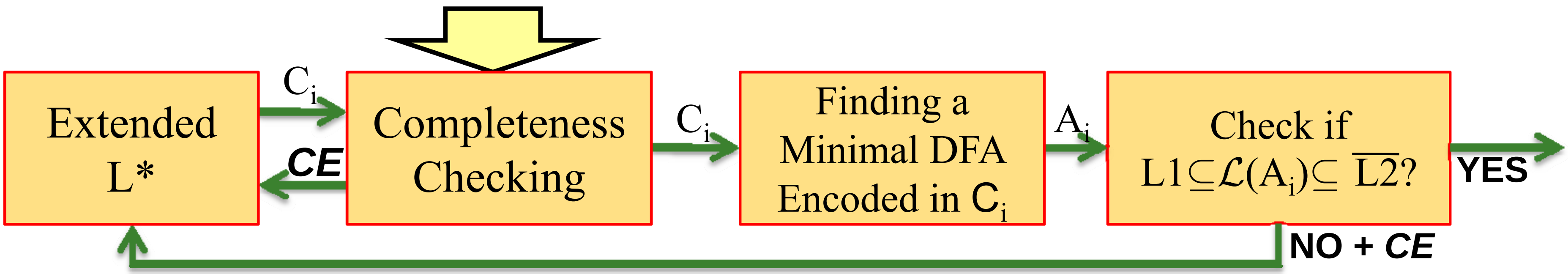


Definition:

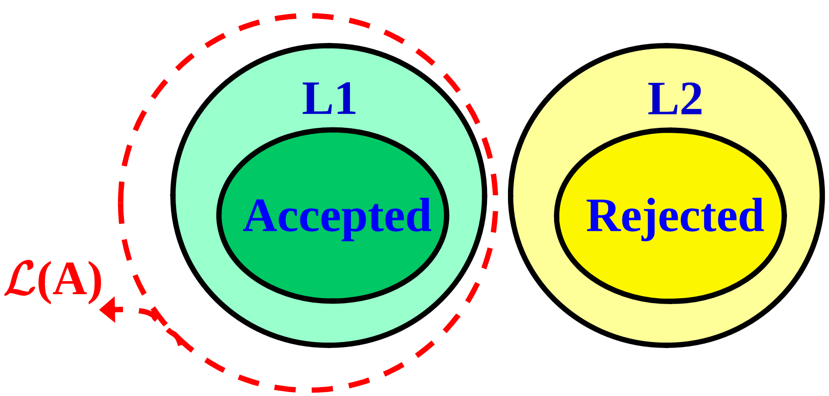
- A **DFA *A*** is **encoded in** a **3DFA *C*** iff ***A***
  - accepts all strings that ***C*** accepts and
  - rejects all strings that ***C*** rejects.
- A **DFA *A*** **separates** **L1** and **L2** iff ***A***
  - accepts all strings in **L1** and
  - rejects all strings in **L2**.

- A minimal DFA encoded in ***C*** is a minimal separating DFA of **L1** and **L2**.

The L^{Sep} Algorithm



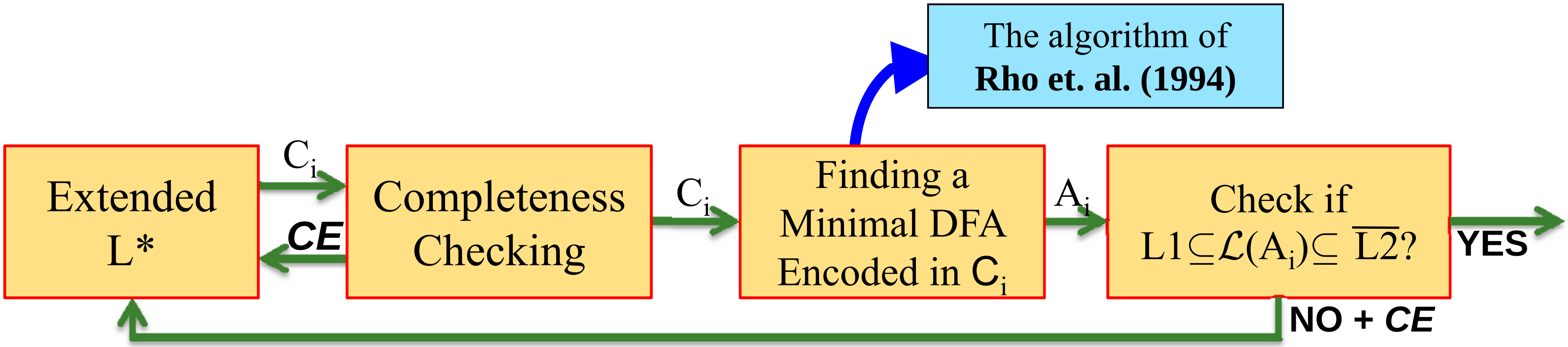
Check if all of the **separating** DFA's of L1 and L2 are **encoded** in C_i, which can be done by checking the following conditions:



Definition:

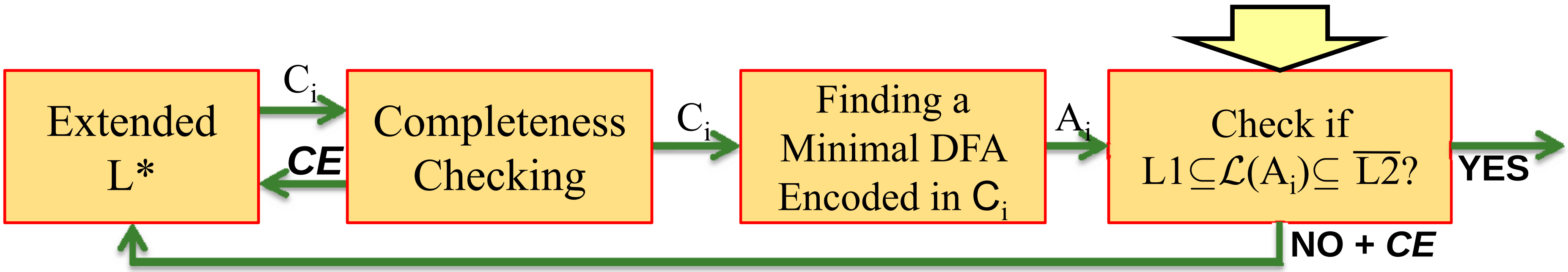
- A **DFA *A*** is **encoded in** a **3DFA *C*** iff ***A***
  - accepts all strings that ***C*** accepts and
  - rejects all strings that ***C*** rejects.
- A **DFA *A*** **separates L1** and **L2** iff ***A***
  - accepts all strings in **L1** and
  - rejects all strings in **L2**.

The L^{Sep} Algorithm



LEMMA:
The size of **minimal separating DFA** of L1 and L2 $\geq$ $|A_i|$, the size of the **minimal DFA encoded in C_i**.

The L^{Sep} Algorithm



If $L1 \subseteq \mathcal{L}(A_i) \subseteq \overline{L2}$:

A_i is a minimal separating DFA

If $L1 \not\subseteq \mathcal{L}(A_i)$ or $\mathcal{L}(A_i) \not\subseteq \overline{L2}$:

The counterexample CE is a witness for C_i is not the target 3DFA

LEMMA:
The size of **minimal separating DFA** of L1 and L2 $\geq$ $|A_i|$, the size of the **minimal DFA encoded in C_i**.

The Algorithm of Gupta *et al.*

Begin with an empty **sample set**

Requires an exponential number of iterations in the worst case

Make a minimal DFA that consistent with the current sample set

A_i

Check if $L1 \subseteq \mathcal{L}(A_i) \subseteq \overline{L2}$?

YES

NO + *CE*

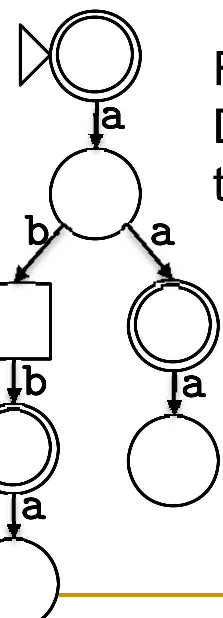
Add ***CE*** to the sample set

An Example

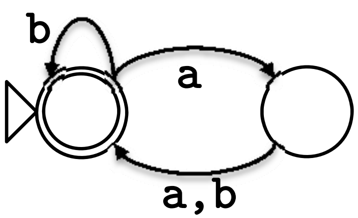
+ SAMPLES:
λ, aa, abb

- SAMPLES:
a, aaa, abba

Make a 3DFA

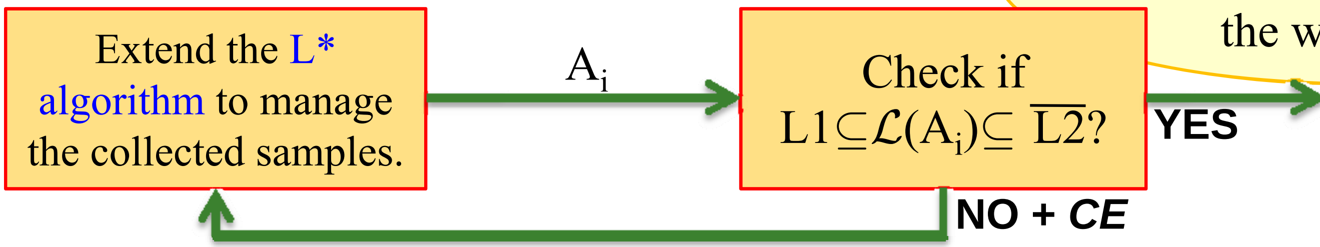


Find a minimal DFA encoded in the 3DFA (NP-hard)



The L^{Sep} Algorithm

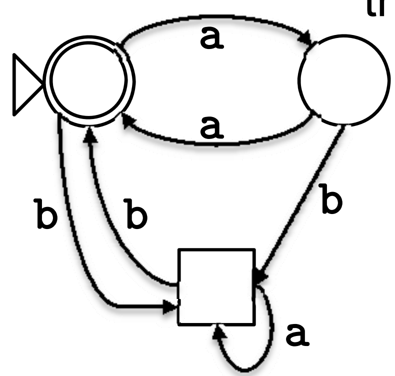
Requires a polynomial number of iterations in the worst case



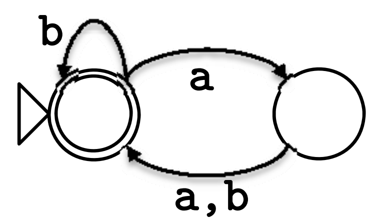
An Example

	λ	a
λ	T	F
a	F	T
ab	D	D
b	D	D
aa	T	F
aba	D	D
abb	T	F

Make a 3DFA



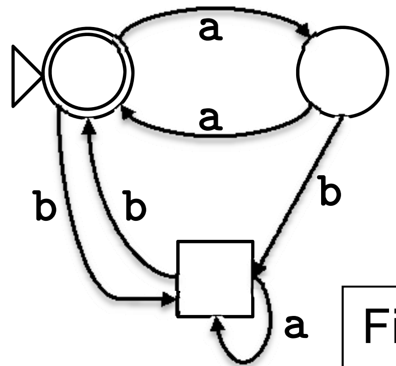
Find a minimal DFA encoded in the 3DFA (NP-hard)

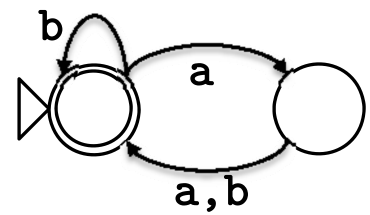


Comparing the Two Algorithms

L^{Sep}:

	λ	a
λ	T	F
a	F	T
ab	D	D
b	D	D
aa	T	F
aba	D	D
abb	T	F





Make a 3DFA

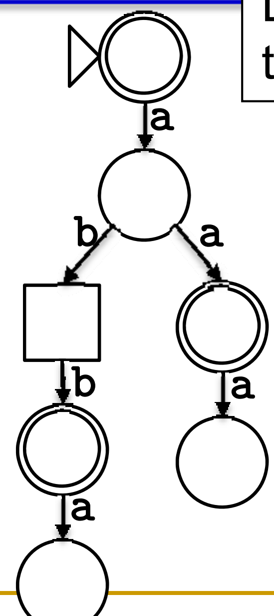
Find a minimal DFA encoded in the 3DFA (NP-hard)

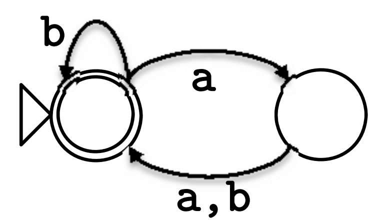
Same sample set!

Gupta *et al.* :

+ SAMPLES:
`λ,aa,abb`

- SAMPLES:
`a,aaa,abba`



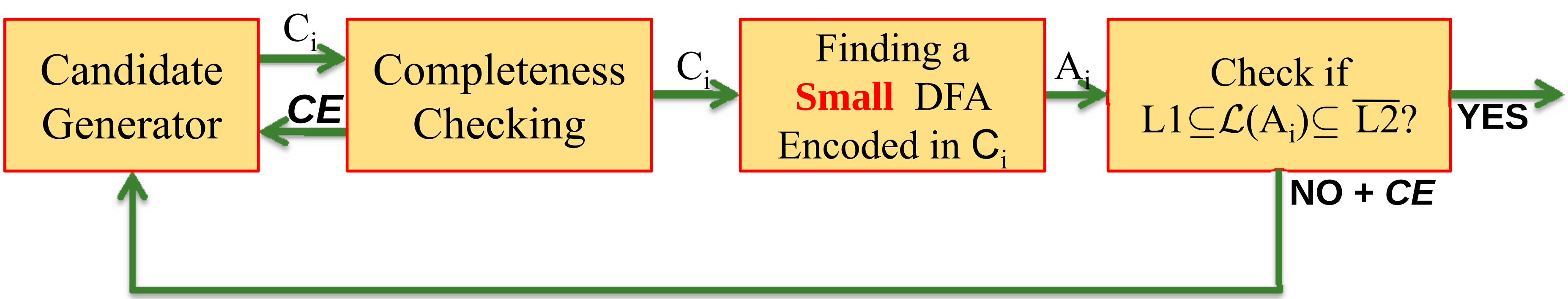


Adapt L^{Sep} for Compositional Verification

- Let $L1 = M2$ and $\overline{L2} = P \cup \overline{M1}$, use L^{Sep} to find a separating DFA for L1 and L2.
- When $M2 \not\subseteq P \cup \overline{M1}$ ($M1 \cap M2 \not\subseteq P$), L^{Sep} can be modified to guarantee finding a string in $M1 \cap M2 \setminus P$.

Adapt L^{Sep} for Compositional Verification

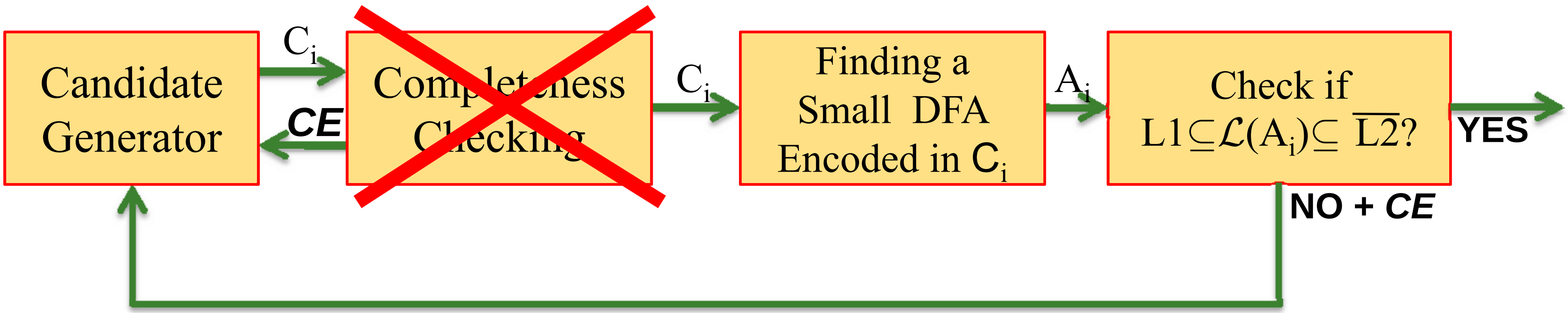
- Use heuristics to find a small consistent DFA:



Minimality is no longer guaranteed!

Adapt L^{Sep} for Compositional Verification

- Skip completeness checking:



Minimality is no longer guaranteed!