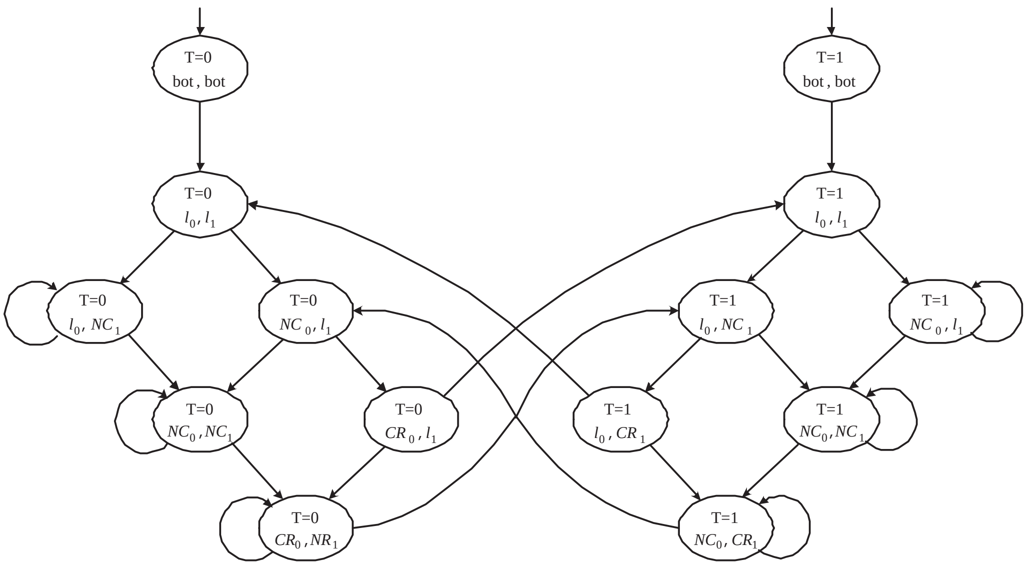
Homework Assignment #1

Note

This assignment is due 9:10AM Wednesday, April 21, 2010. Please write or type your answers on A4 (or similar size) paper. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

1. (20 points) Consider model checking the CTL property $\mathbf{AG}(l_0 \rightarrow \mathbf{AF}\,CR_0)$ (using the procedures in Chapter 4.1 of [CGP 1999]) against the following Kripke structure for a two-process mutual exclusion program. Note that we are treating the statement labels l_0 and CR_0 as atomic propositions.



(Source: redrawn from [CGP 1999, Fig 2.2])

Please illustrate the steps of labeling the states with sub-formulae during the execution of the model checking algorithm. As you will see, the property does not hold (i.e., there is possibility of starvation). What fairness constraints should be added?

2. (20 points) The following is a NuSMV model for two asynchronous processes that use a semaphore to achieve mutual exclusion.

```
MODULE main
VAR
  semaphore : boolean;
  proc1     : process user(semaphore);
  proc2     : process user(semaphore);
ASSIGN
  init(semaphore) := 0;

MODULE user(semaphore)
VAR
  state : {idle, entering, critical, exiting};
ASSIGN
  init(state) := idle;
  next(state) :=
    case
      state = idle                  : {idle, entering};
      state = entering & !semaphore : critical;
      state = critical              : {critical, exiting};
      state = exiting               : idle;
      1                             : state;
    esac;
  next(semaphore) :=
    case
      state = entering : 1;
      state = exiting  : 0;
      1                : semaphore;
    esac;
```

(a) Write all the necessary boolean formulae that specify the main module as a Kripke structure; you may define shorter substitute names for the variables to save space.

(b) Please draw BDD diagrams (as small as possible) for the formulae in 2a.

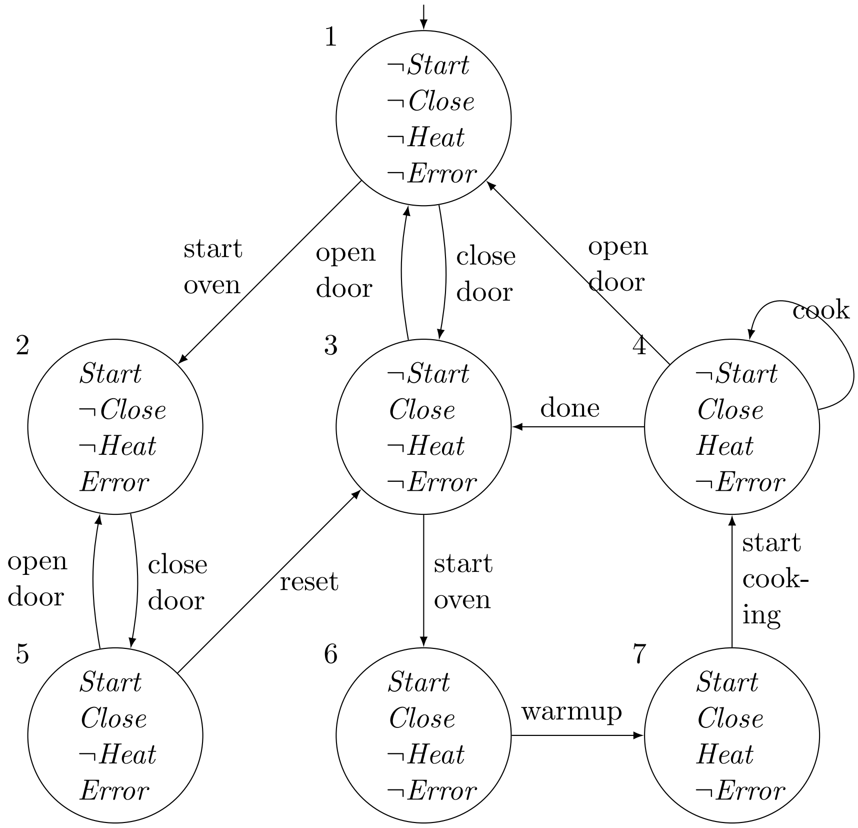
3. (10 points) For an ordered set of your choice, find a self-map on the set (i.e., a function mapping from the set to itself) that is monotonic (order-preserving), but not $\cup$-continuous. Please define monotonicity and $\cup$-continuity precisely in terms of the chosen ordered set before presenting the example self-map.

4. (30 points) Consider symbolic model checking of CTL on finite Kripke structures. Prove that, for any CTL formula f, the following statements hold:

(a) The set of states satisfying $\mathbf{AF}f$ is the least fixpoint of the function $\tau(Z) = f \vee \mathbf{AX}Z$.

(b) The set of states satisfying $\mathbf{AG}f$ is the greatest fixpoint of the function $\tau(Z) = f \wedge \mathbf{AX}Z$.

5. (20 points) The microwave oven example in [CGP] is redrawn as follows.



Please use the symbolic LTL model checking algorithm in [CGP; Chapter 6] to verify if $\mathbf{GF}\,\mathit{Close}$ is valid (i.e., holds for all paths) in this system. You may define shorter substitute names for the propositions to save space.