Compositional Reasoning

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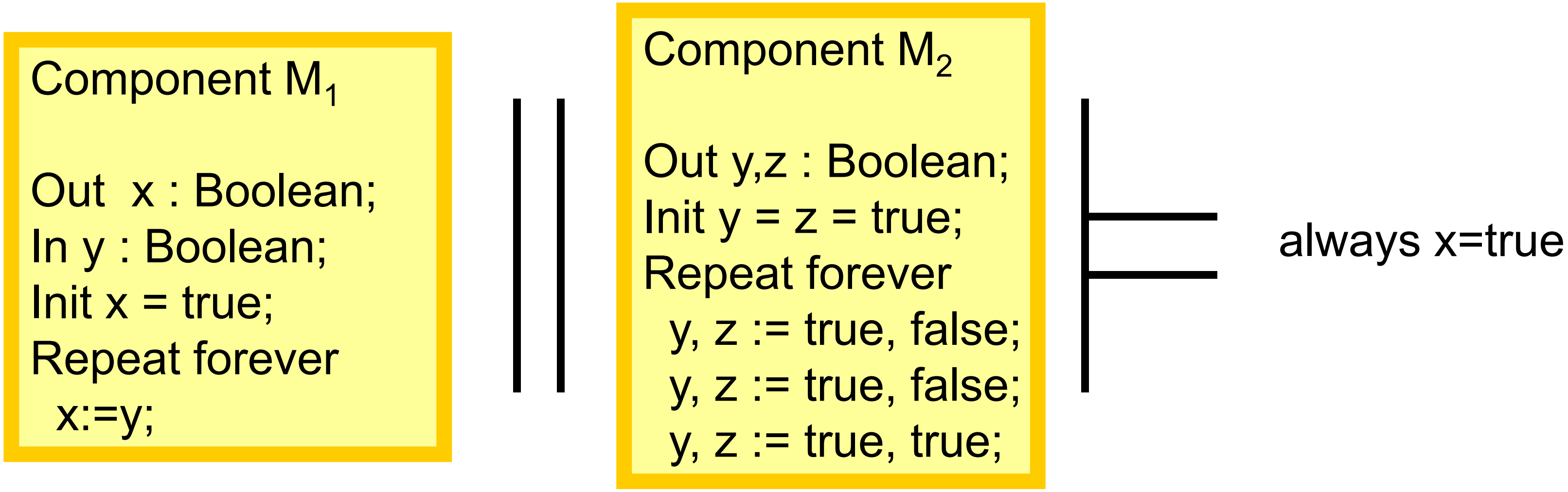
Verification of Parallel Compositions

- **Verification Task:** verify if the system composed of components M_1 and M_2 satisfies a property P, i.e., $M_1 || M_2 \models P$.
- M_1 and M_2 may rely on each other to satisfy P.



Component M_1

```
Out  x : Boolean;
In y : Boolean;
Init x = true;
Repeat forever
  x:=y;
```

||

Component M_2

```
Out y,z : Boolean;
Init y = z = true;
Repeat forever
  y, z := true, false;
  y, z := true, false;
  y, z := true, true;
```

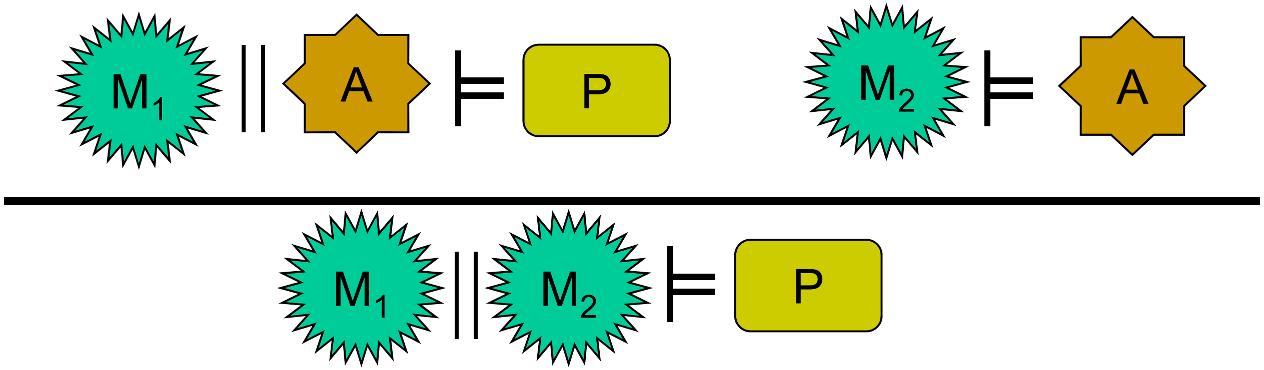
$\models$ always x=true

M_1 alone does not guarantee "always x = true"!

- Can the construction of $M_1 || M_2$ be avoided?

Compositional Reasoning

- An **Assume-Guarantee** (A-G) rule:



- If a small *contextual assumption* A (an abstraction of M_2) exists, then the overall verification task may become easier.

But, how to find A automatically?

- It is possible when M_1, M_2, A, and P are finite automata.

Compositional Reasoning (cont.)

```
Component $M_1$

Out  x : Boolean;
In y,z : Boolean;
Init x = true;
Repeat forever
  x := y;
```

$\|$

```
Component $M_2$

Out y,z : Boolean;
Init y = z = true;
Repeat forever
  y, z := true, false;
  y, z := true, false;
  y, z := true, true;
```

$\models$ always x=true

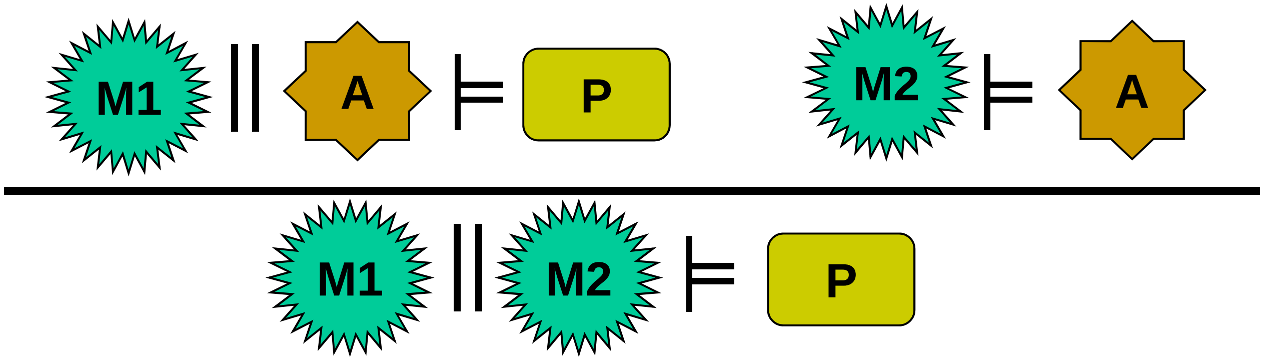
A suitable contextual assumption A :

```
Component A

Out y,z : Boolean;
Init y = true;
Repeat forever
  y, z := true, ?;
```

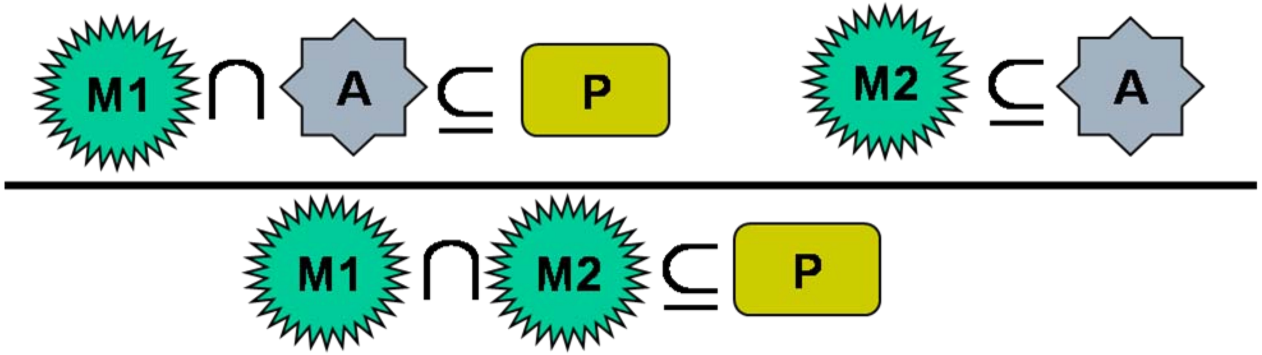
Component A has fewer states (automaton locations) than M_2.

Setting the Stage



$$\frac{M1 \parallel A \models P \qquad M2 \models A}{M1 \parallel M2 \models P}$$

- ☐ The **behaviors of components** and **properties** are described as **regular languages.**
- ☐ **Parallel composition** is presented by **the intersection of the languages.**
- ☐ **A system satisfies a property** if **the language of the system is a subset of the language of the property.**

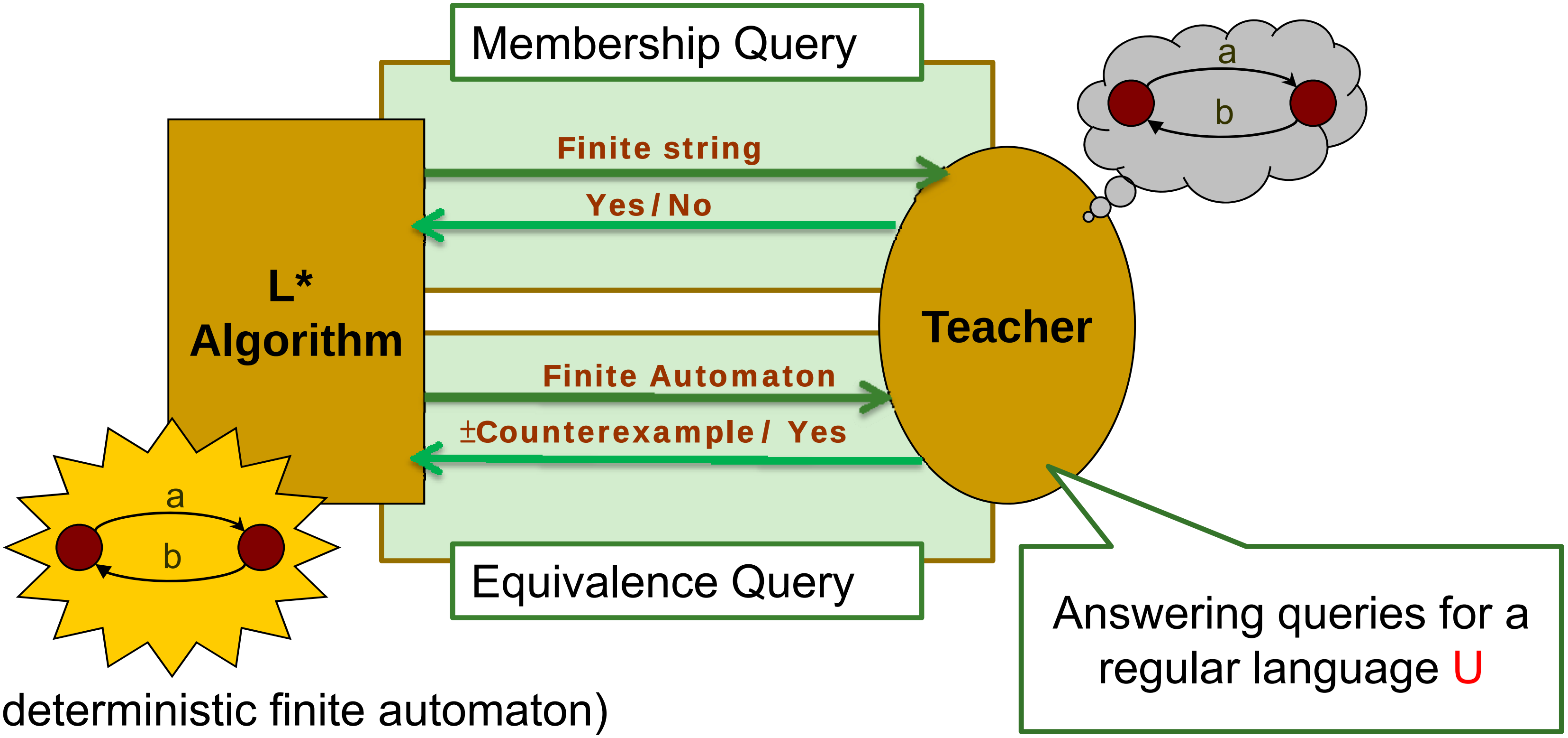


$$\frac{M1 \cap A \subseteq P \qquad M2 \subseteq A}{M1 \cap M2 \subseteq P}$$

Outline

- Learning-Based Compositional Model Checking:
    - Automation by Learning
    - The L* Algorithm
    - The Problem of L*-Based Approaches
- Learning Minimal Separating DFA's:
    - The L^{SEP} Algorithm
    - Comparison with Another Algorithm
    - Adapt L^{SEP} for Compositional Model Checking

Overview of the L* Algorithm



Membership Query

Finite string

Yes / No

L* Algorithm

Teacher

Finite Automaton

±Counterexample / Yes

Equivalence Query

DFA (deterministic finite automaton)

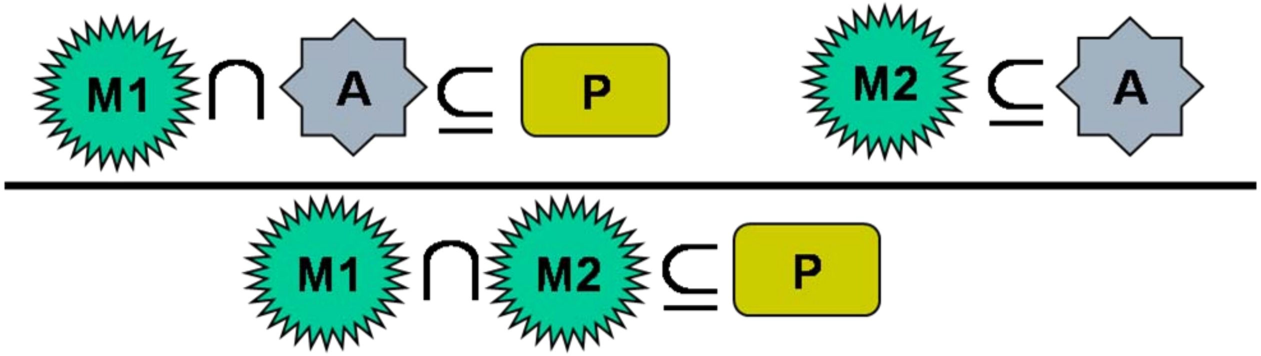
Answering queries for a regular language U

If such a teacher is provided, L* guarantees to produce a DFA that recognizes U using a polynomial number of queries.

Automation by Learning

- **First developed** by Cobleigh, Giannakopoulou, and Păsăreanu [TACAS 2003]

- Apply the L^* learning algorithm for regular languages to find an A for the A-G rule:



Basic Understanding

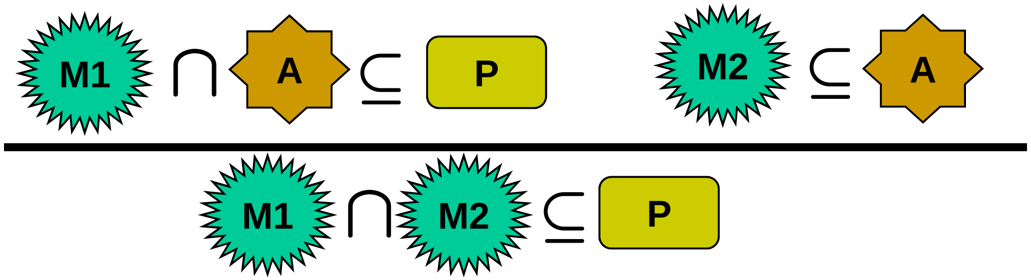
- A closer look at the A-G rule:

$M1\cap A\subseteq P \Leftrightarrow$

$M1\cap A\cap\overline{P}=\emptyset\Leftrightarrow$

$A\cap\overline{(P\cup\overline{M1})}=\emptyset \Leftrightarrow$

$A\subseteq P\cup\overline{M1}$



When $A=P\cup\overline{M1}$,

$M2\subseteq A\Leftrightarrow$

$M2\subseteq P\cup\overline{M1}\Leftrightarrow$

$M2\cap\overline{(P\cup\overline{M1})}=\emptyset\Leftrightarrow$

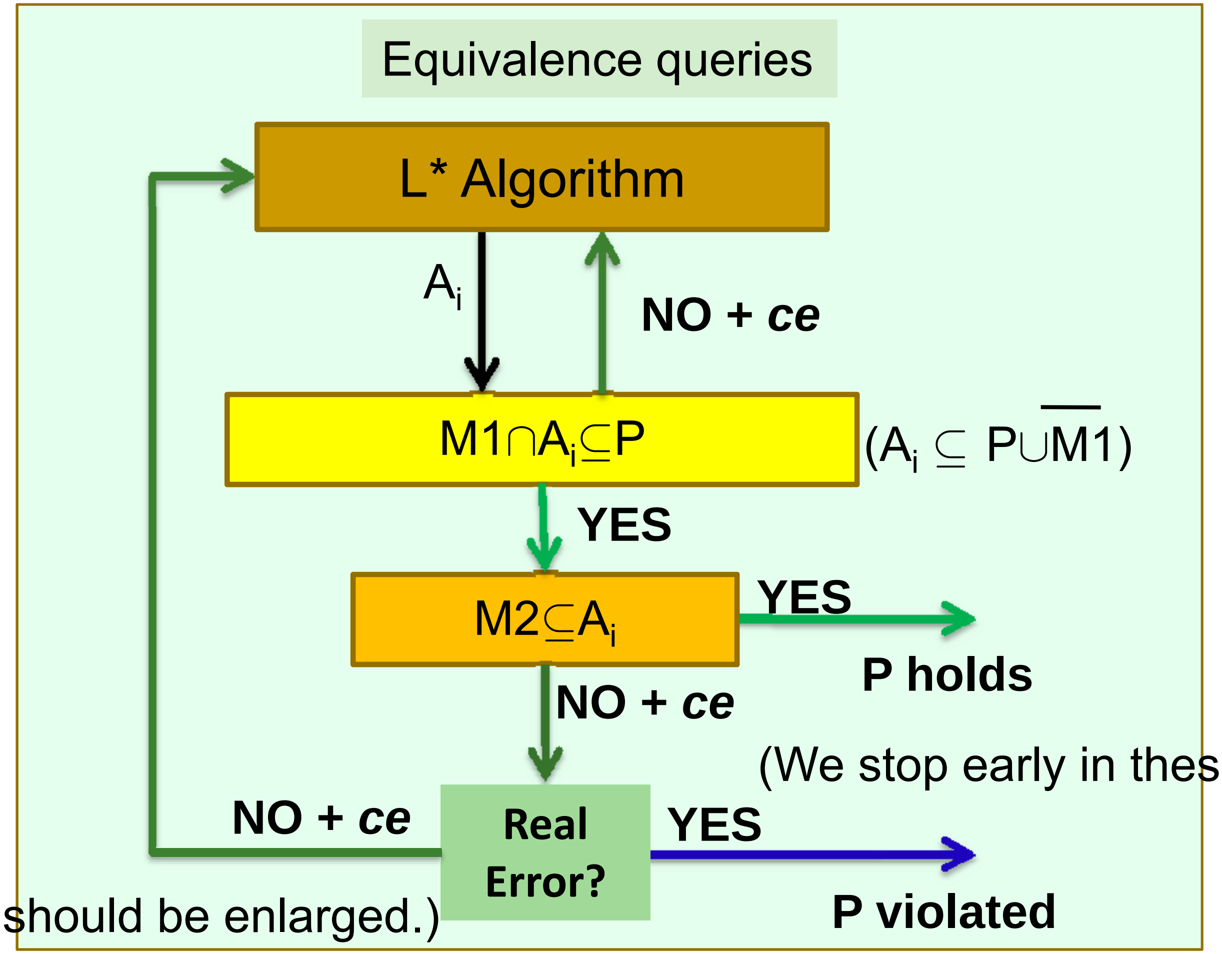
$M2\cap\overline{P}\cap M1=\emptyset\Leftrightarrow$

$M1\cap M2\subseteq P$

- Conceptually, the target language is **$P\cup\overline{M1}$**, the *weakest assumption* for the premise **$M1\cap A\subseteq P$**.
- Actually reaching the target would be even worse than checking **$M1\cap M2\subseteq P$** directly.
- It really pays off when we can stop earlier …

The Algorithm of Cobleigh *et al.*

Target: $P \cup \overline{M1}$



Equivalence queries

L* Algorithm

A_i

NO + *ce*

$M1 \cap A_i \subseteq P$

$(A_i \subseteq P \cup \overline{M1})$

YES

$M2 \subseteq A_i$

YES

P holds

NO + *ce*

(We stop early in these two cases.)

NO + *ce*

Real Error?

YES

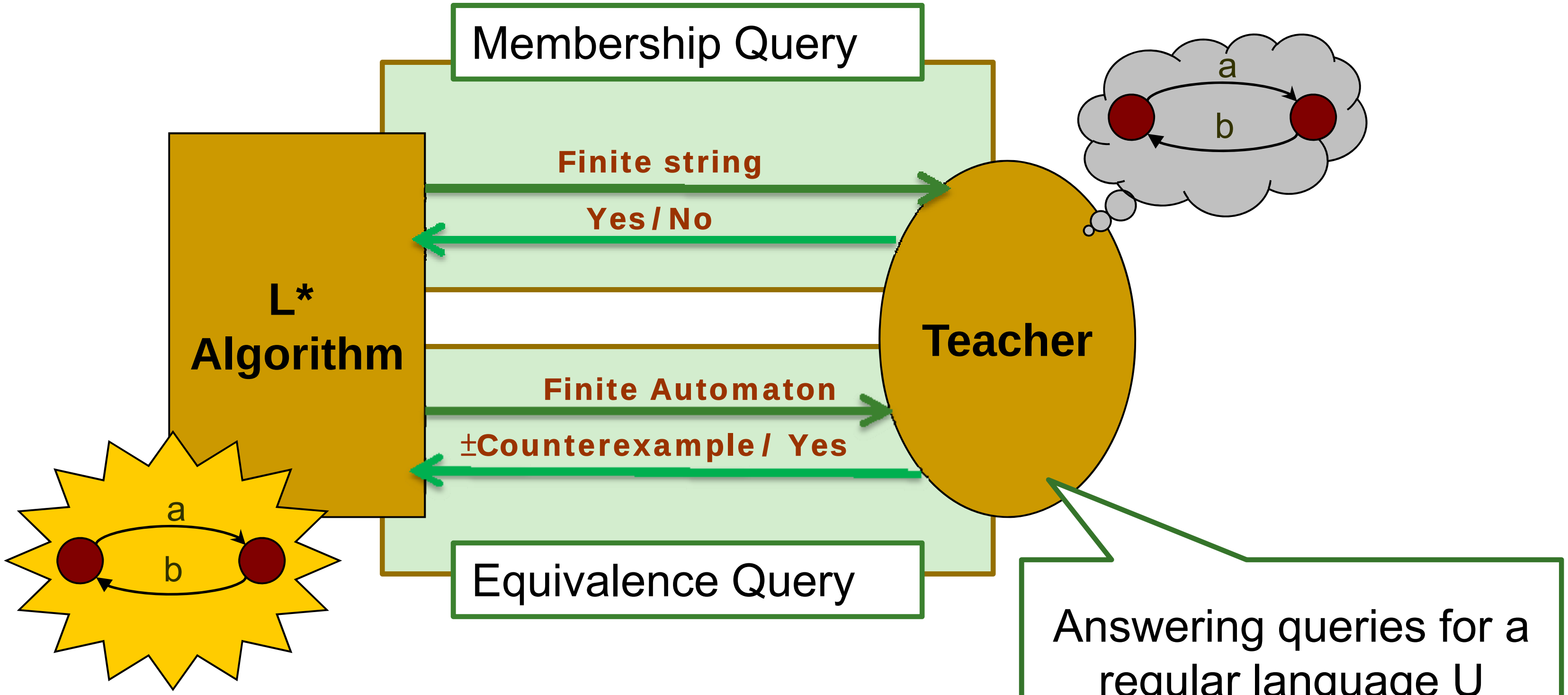
P violated

(*ce* tells how A_i should be enlarged.)

(*ce* is a real error if *ce* is in M2, but not in $P \cup \overline{M1}$, implying $M2 \not\subseteq P \cup \overline{M1}$, i.e., $M1 \cap M2 \not\subseteq P$.)

The L* Learning Algorithm

- Proposed by D. Angluin [Info.&Comp. 1987] and improved by Rivest and Schapire [Info.&Comp. 1993]



Membership Query

Finite string

Yes / No

L* Algorithm

Teacher

Finite Automaton

±Counterexample / Yes

Equivalence Query

Answering queries for a regular language U

DFA (deterministic finite automaton)

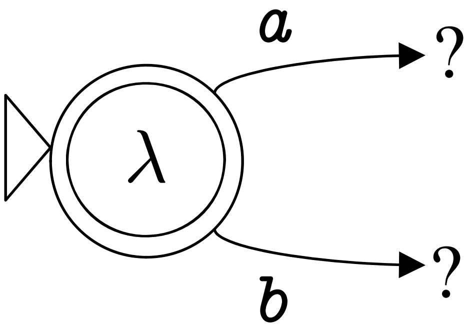
L*: Initial Setting

E: Distinguishing Experiments

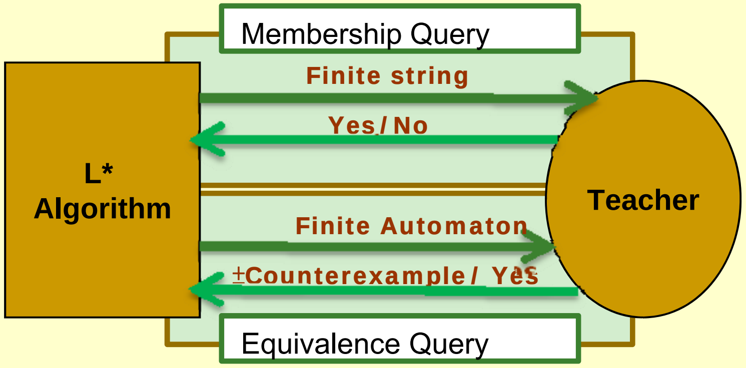
States S

Next States SΣ

	λ
λ	
a	
b	



A reminder:



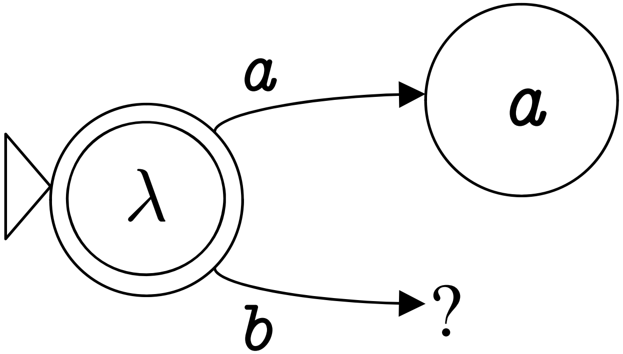
Target: $(ab+aab)^*$

L*: Fill Up the Table by Membership Queries

	λ
λ	T
a	F
b	F

Fill up the table using **membership queries**.

a represents a new equivalence class, because its **row** is different from all of those in the current S set.



Target: $(ab+aab)^*$

L*: Table Expansion

Move a to the S set and expand the table with elements aa and ab.

	λ
λ	T
a	F
b	F
aa	
ab	

Target: $(ab+aab)^*$

L*: A Closed Table

	λ
λ	T
a	F
b	F
aa	F
ab	T

Again, fill up the table using membership queries.

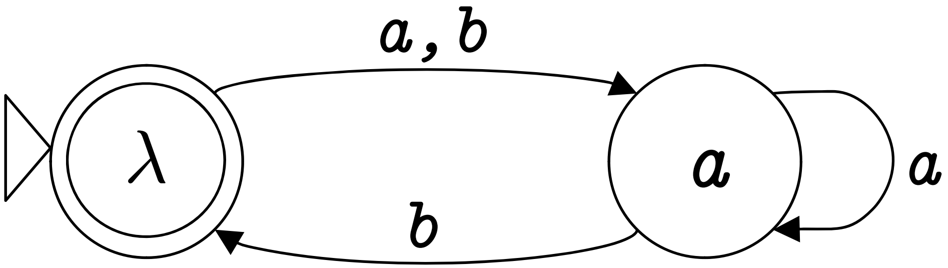
We say that the table is **closed** because every row in the SΣ set appears somewhere in the S set.

Target: $(ab+aab)^*$

L*: Making a Conjecture

	λ
λ	T
a	F
b	F
aa	F
ab	T

Construct a DFA from the learned equivalence classes.



Counterexample: bb

≠

$\delta(s, a) = s'$ iff sa and s' have the same row.

A suffix b is extracted from bb as a valid distinguishing experiment

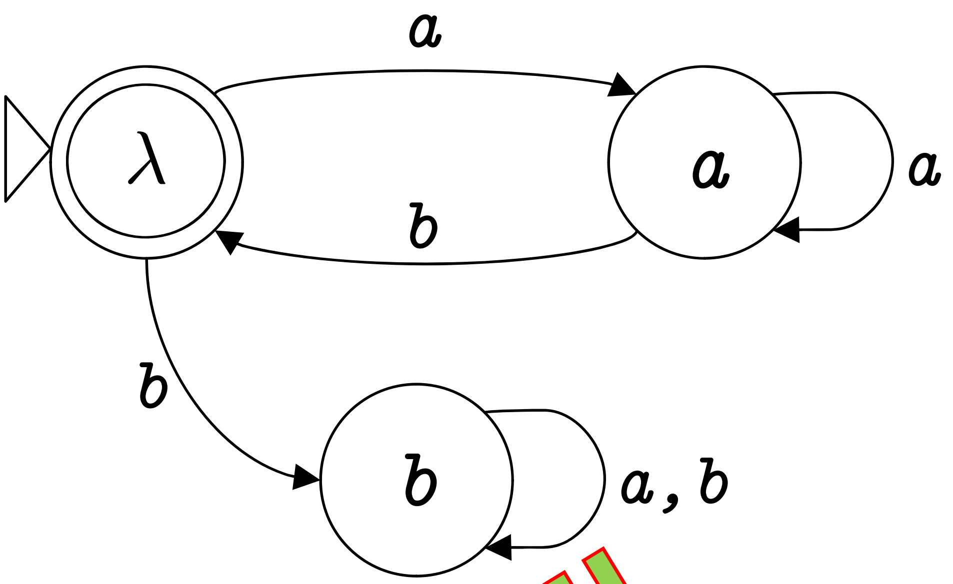
Target: $(ab+aab)^*$

Theorem:
At least one suffix of the counterexample is a valid distinguishing experiment.

L*: 2nd Iteration

Add b to the E set, fill up and expand the table following the same procedure.

	λ	b
λ	T	F
a	F	T
b	F	F
aa	F	T
ab	T	F
ba	F	F
bb	F	F



Counterexample: $aaab$

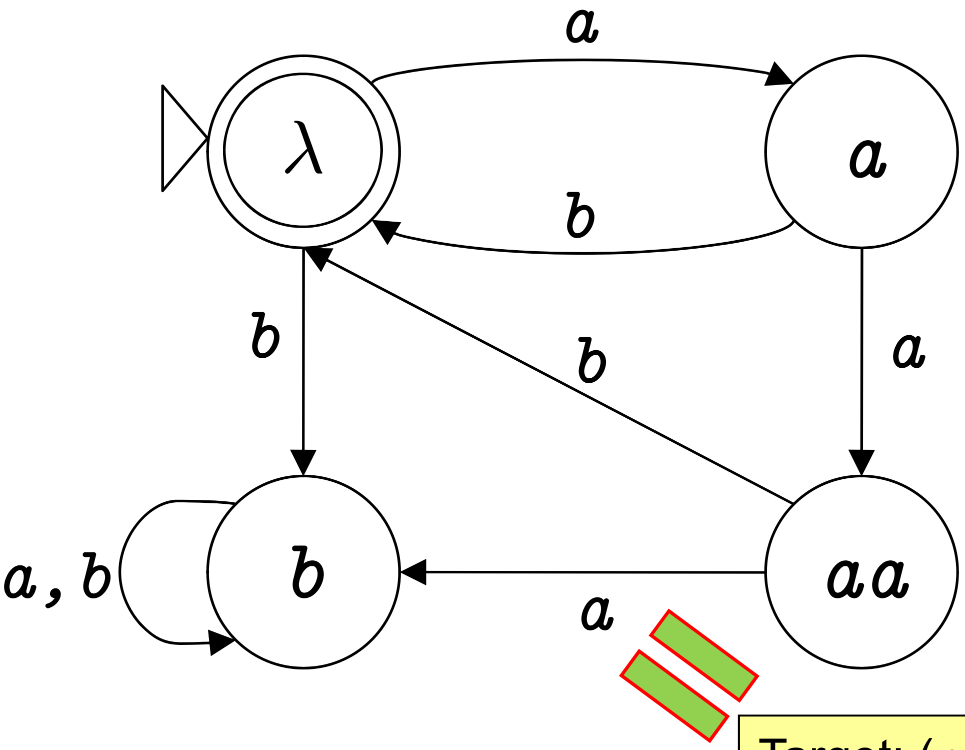
A suffix ab is extracted from $aaab$ as a valid distinguishing experiment.

Target: $(ab+aab)^*$

L*: 3rd Iteration (Completed)

Add *ab* to the E set, fill up and expand the table following the same procedure.

	λ	b	ab
λ	T	F	T
a	F	T	T
b	F	F	F
aa	F	T	F
ab	T	F	T
ba	F	F	F
bb	F	F	F
aaa	F	F	F
aab	T	F	T



Target: $(ab+aab)^*$

Theorem:
The DFA produced by L* is the minimal DFA that recognizes that target language.

L*: Complexity

- **Complexity:**
  - Equivalence query: at most n
  - Membership query: $O(|\Sigma|n^2 + n \log m)$

	λ	b	ab
λ	T	F	T
a	F	T	T
b	F	F	F
aa	F	T	F
ab	T	F	T
ba	F	F	F
bb	F	F	F
aaa	F	F	F
aab	T	F	T

Note: ***n*** is the size of the minimal DFA that recognizes U, ***m*** is the length of the longest counterexample returned from the teacher.

The Problem

- The L*-based approaches cannot guarantee finding the **minimal assumption** (in size), even if there exists one.

$$\frac{M1 \cap A \subseteq P \qquad M2 \subseteq A}{M1 \cap M2 \subseteq P}$$

- The smaller the size of A is, the easier it is to check the correctness of the two premises.

- L^* targets a single language, however, there exists a range of languages that satisfy the premises of an A-G rule.

Finding a Minimal Assumption

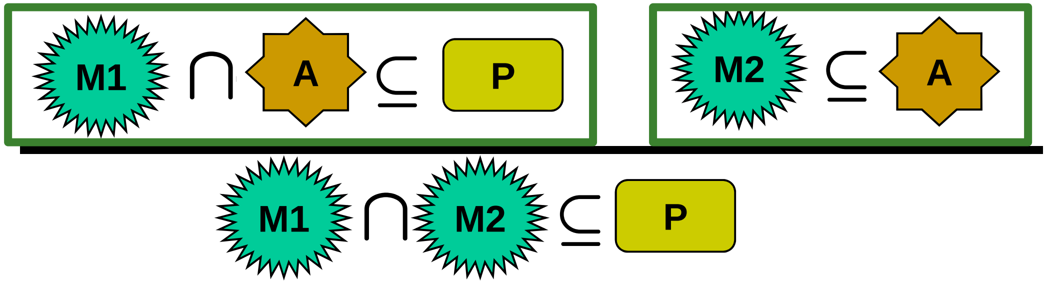
- **A reminder**: we use the following Assume-Guarantee rule for decomposition.

$M1 \cap A \subseteq P \Leftrightarrow$

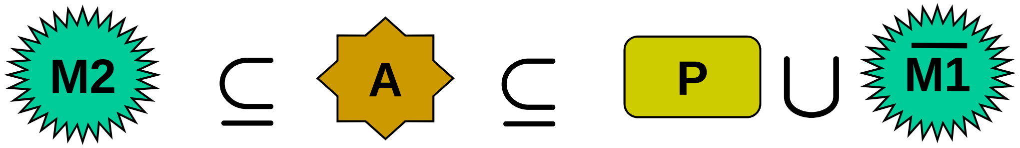
$M1 \cap A \cap \overline{P} = \emptyset \Leftrightarrow$

$A \cap \overline{(P \cup \overline{M1})} = \emptyset \Leftrightarrow$

$A \subseteq P \cup \overline{M1}$



- The two premises can be rewritten as follows:



Finding a Minimal Assumption (cont.)

- To apply the A-G rule is to find an A satisfying the following constraint:

$$M2 \subseteq A \subseteq P \cup \overline{M1}$$

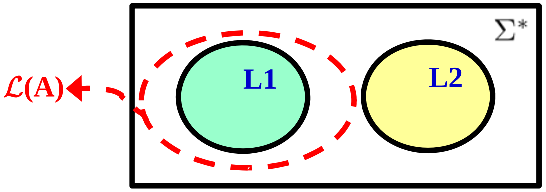
- So, **the problem of finding a minimal assumption** for the A-G rule reduces to **finding a minimal separating DFA** that
  - **accepts** every string in **M2** and
  - **rejects** every string not in **P** ∪ $\overline{\textbf{M1}}$.

First observed by Gupta, McMillan, and Fu

Learning a Minimal Separating DFA

- **Contribution of [Chen et al. TACAS 2009]:** a **polynomial-query** learning algorithm, L^{Sep}, for minimal separating DFA's.
- **Problem:** given two disjoint regular languages **L1** and **L2**, we want to find a minimal DFA **A** that satisfies

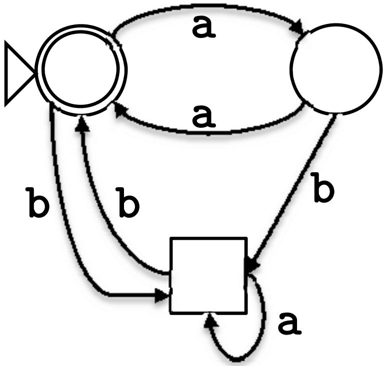
$$L1 \subseteq \mathcal{L}(A) \subseteq \overline{L2}$$



We say that **A** is a separating DFA for L1 and L2

- **Assumption:** a teacher for L1 and L2:
  - Membership query: if a string **s** is in L1 (resp. L2)
  - Containment query: $? \subseteq L1$, $? \supseteq L1$, $? \subseteq L2$, and $? \supseteq L2$

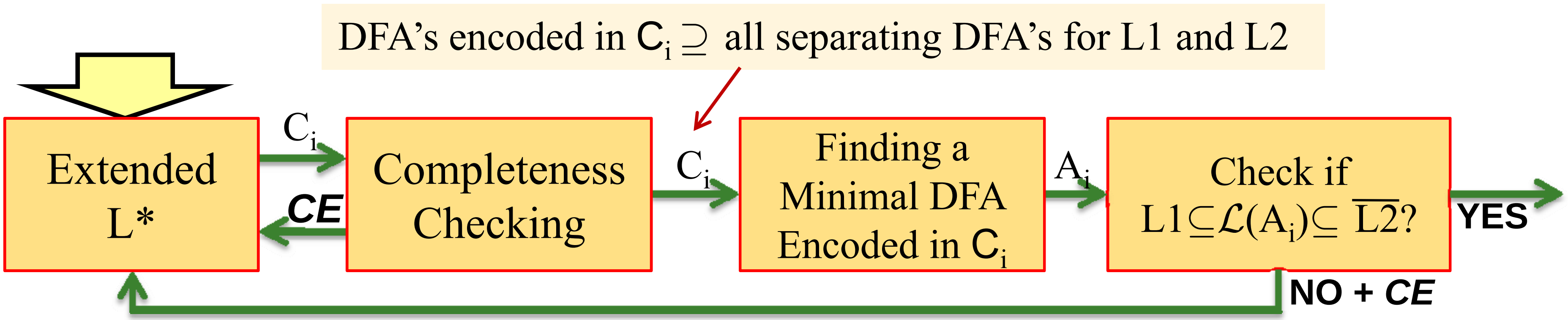
3-Value DFA (3DFA)



An example of a 3DFA

- A 3DFA is a tuple $\mathcal{C} = (\Sigma, S, s_0, \delta, Acc, Rej, Dont)$.
- A **DFA** ***A*** is **encoded in** a **3DFA** ***C*** iff ***A***
  - accepts all strings that ***C*** accepts and
  - rejects all strings that ***C*** rejects.
  - A don't care string in ***C*** can be either accepted or rejected by ***A***.

The L^{Sep} Algorithm: Overview



DFA's encoded in $C_i \supseteq$ all separating DFA's for L1 and L2

Extended L*

C_i

CE

Completeness Checking

C_i

Finding a Minimal DFA Encoded in C_i

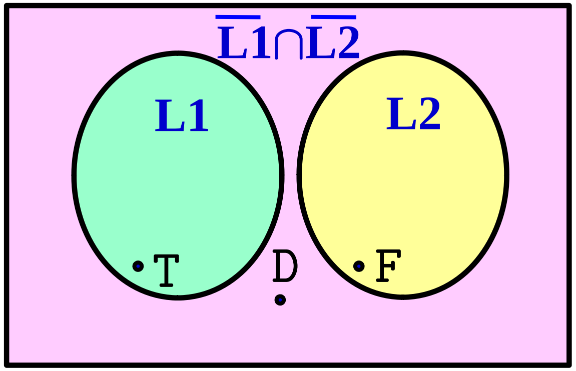
A_i

Check if $L1 \subseteq \mathcal{L}(A_i) \subseteq \overline{L2}$?

YES

NO + *CE*

Target:



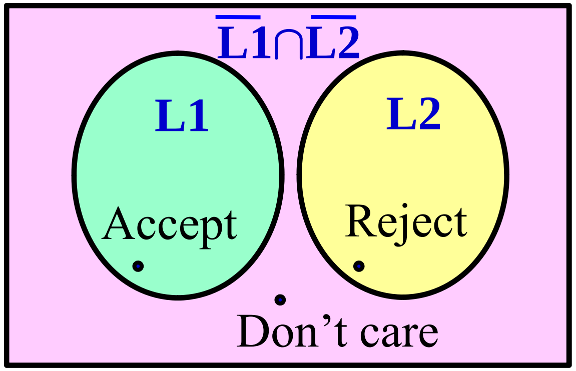
	λ	a
λ	T	F
a	F	T
ab	D	D
b	D	D
aa	T	F
aba	D	D
abb	T	F

Extend the **L* algorithm** to allow **don't care** values.

The Target 3DFA

- The target 3DFA ***C***
  - **accepts** every string in **L1**, and
  - **rejects** every string in **L2**.
  - Strings in $\overline{L1} \cap \overline{L2}$ are **don't care** strings.

DFA's encoded in C =
all separating DFA's for L1 and L2

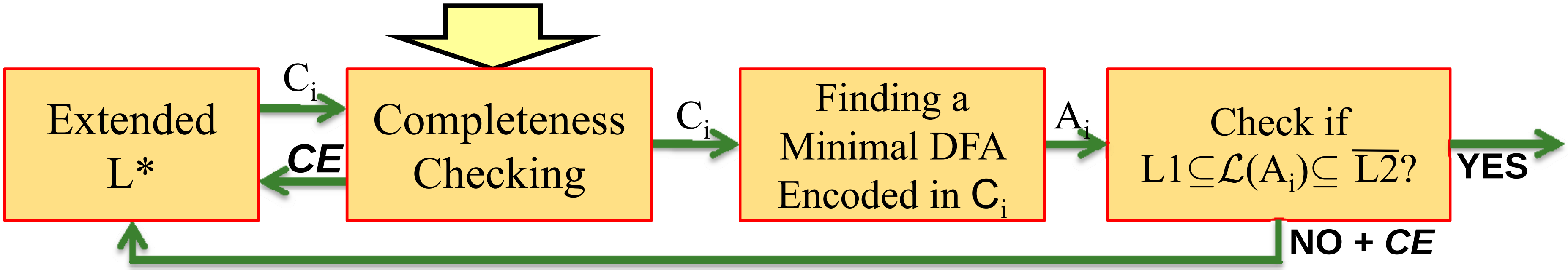


Definition:

- A **DFA *A*** is **encoded in** a **3DFA *C*** iff ***A***
  - accepts all strings that ***C*** accepts and
  - rejects all strings that ***C*** rejects.
- A **DFA *A*** **separates** **L1** and **L2** iff ***A***
  - accepts all strings in **L1** and
  - rejects all strings in **L2**.

- A minimal DFA encoded in ***C*** is a minimal separating DFA of **L1** and **L2**.

The L^{Sep} Algorithm



Extended L*

C_i

CE

Completeness Checking

C_i

Finding a Minimal DFA Encoded in C_i

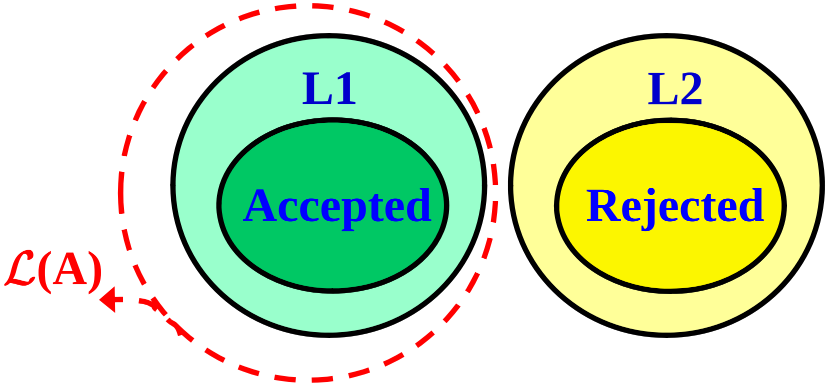
A_i

Check if $L1 \subseteq \mathcal{L}(A_i) \subseteq \overline{L2}$?

YES

NO + *CE*

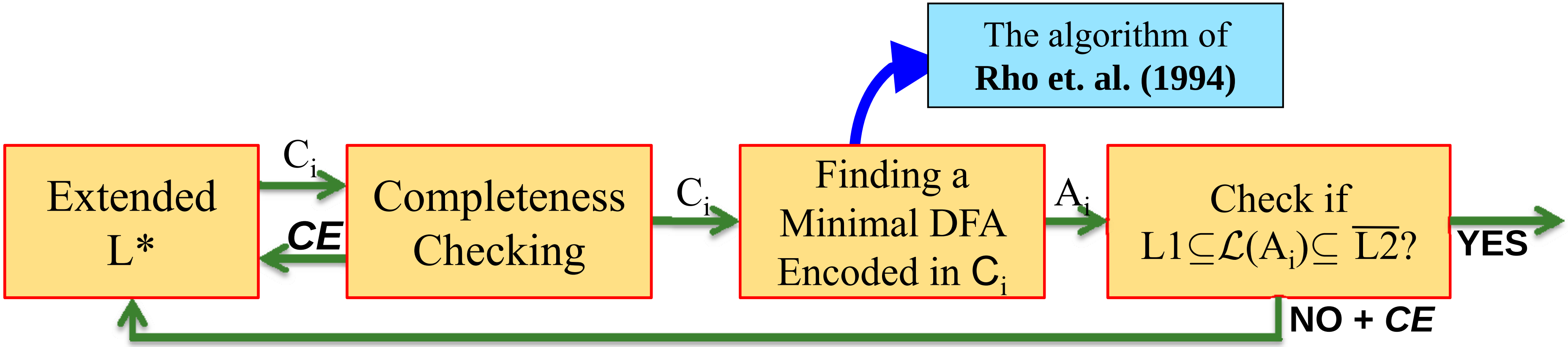
Check if all of the **separating** DFA's of L1 and L2 are **encoded** in C_i, which can be done by checking the following conditions:



Definition:

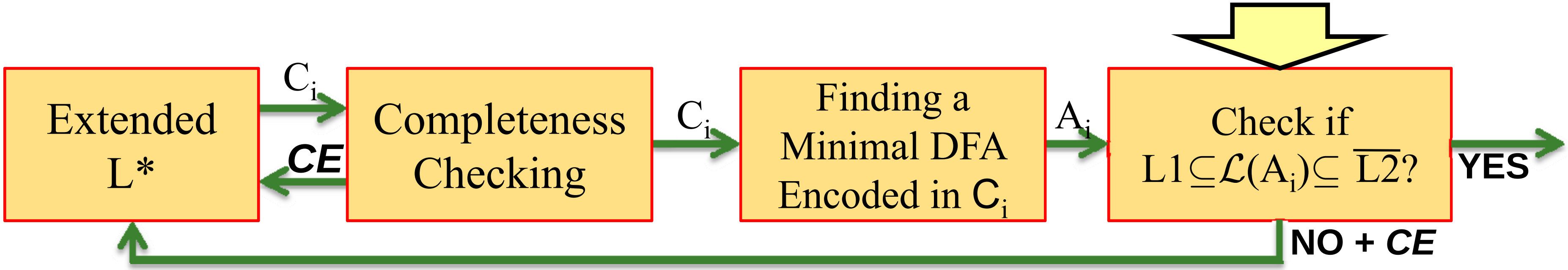
- A **DFA *A*** is **encoded in** a **3DFA *C*** iff ***A***
  - accepts all strings that ***C*** accepts and
  - rejects all strings that ***C*** rejects.
- A **DFA *A* separates L1** and **L2** iff ***A***
  - accepts all strings in **L1** and
  - rejects all strings in **L2**.

The L^{Sep} Algorithm



LEMMA:
The size of **minimal separating DFA** of L1 and L2 $\geq$ $|A_i|$, the size of the **minimal DFA encoded in Ci**.

The L^{Sep} Algorithm



If $L1 \subseteq \mathcal{L}(A_i) \subseteq \overline{L2}$:

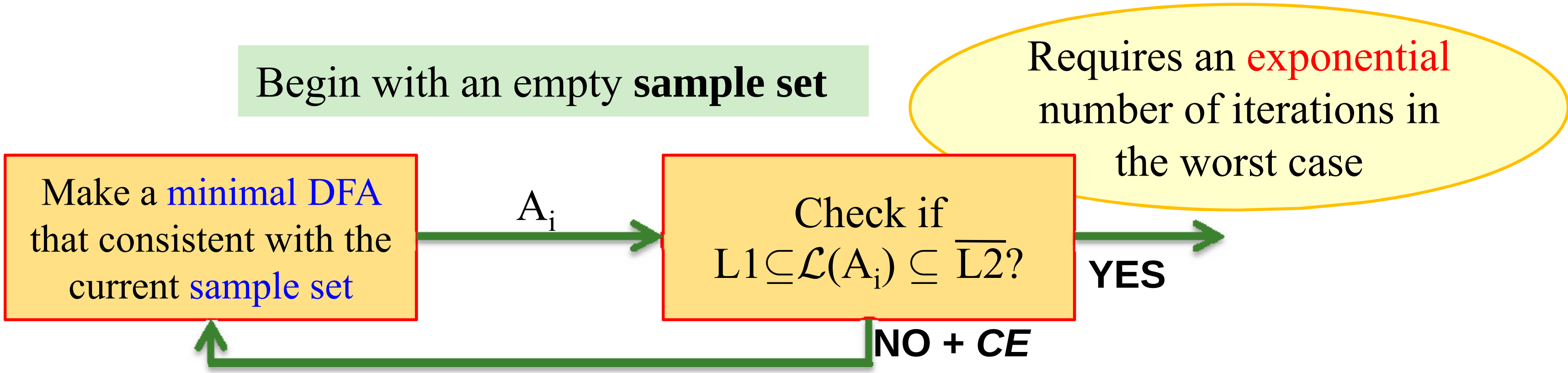
A_i is a minimal separating DFA.

If $L1 \not\subseteq \mathcal{L}(A_i)$ or $\mathcal{L}(A_i) \not\subseteq \overline{L2}$:

Counterexample CE is a witness for C_i not being the target 3DFA.

LEMMA:
The size of **minimal separating DFA** of L1 and L2 $\geq$ $|A_i|$, the size of the **minimal DFA encoded in Ci**.

The Algorithm of Gupta *et al.*



Begin with an empty **sample set**

Requires an exponential number of iterations in the worst case

Make a minimal DFA that consistent with the current sample set

A_i

Check if $L1 \subseteq \mathcal{L}(A_i) \subseteq \overline{L2}$?

YES

NO + *CE*

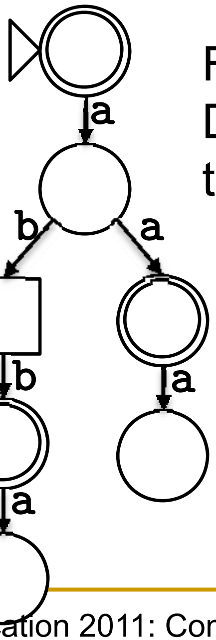
Add ***CE*** to the sample set

An Example

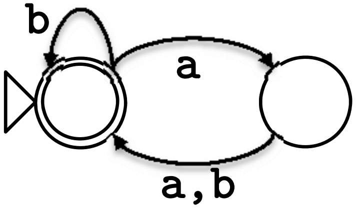
+ SAMPLES:
λ,aa,abb

- SAMPLES:
a,aaa,abba

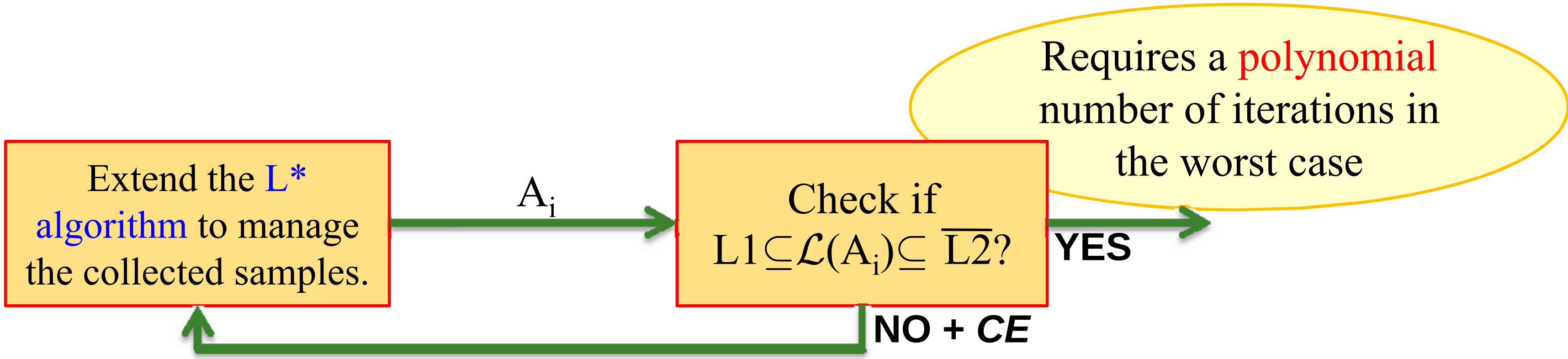
Make a 3DFA



Find a minimal DFA encoded in the 3DFA (NP-hard)



The L^{Sep} Algorithm



Extend the L* algorithm to manage the collected samples.

A_i

Check if $L1 \subseteq \mathcal{L}(A_i) \subseteq \overline{L2}$?

YES

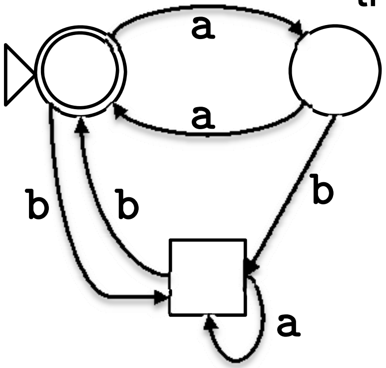
NO + *CE*

Requires a polynomial number of iterations in the worst case

An Example

	λ	a
λ	T	F
a	F	T
ab	D	D
b	D	D
aa	T	F
aba	D	D
abb	T	F

Make a 3DFA



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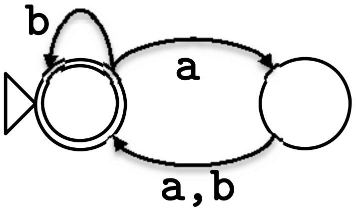
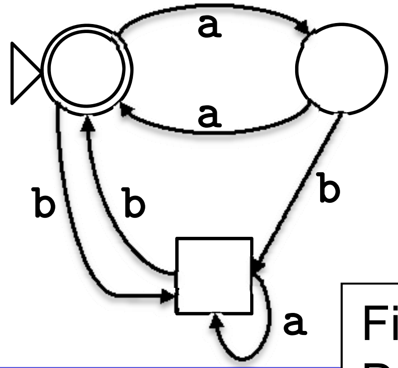


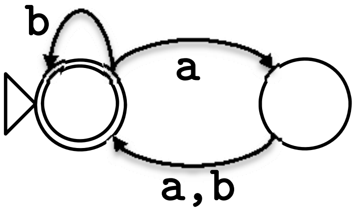
Comparing the Two Algorithms

L^{Sep}:

	λ	a
λ	T	F
a	F	T
ab	D	D
b	D	D
aa	T	F
aba	D	D
abb	T	F

Make a 3DFA





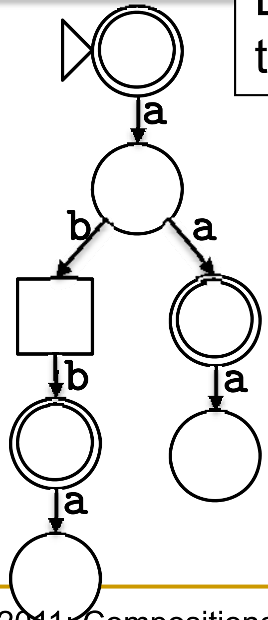
Find a minimal DFA encoded in the 3DFA (NP-hard)

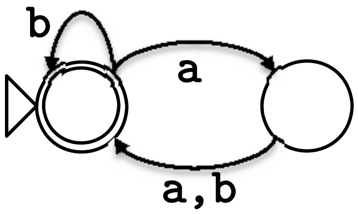
Same sample set!

Gupta *et al.* :

+ SAMPLES:
`λ,aa,abb`

- SAMPLES:
`a,aaa,abba`



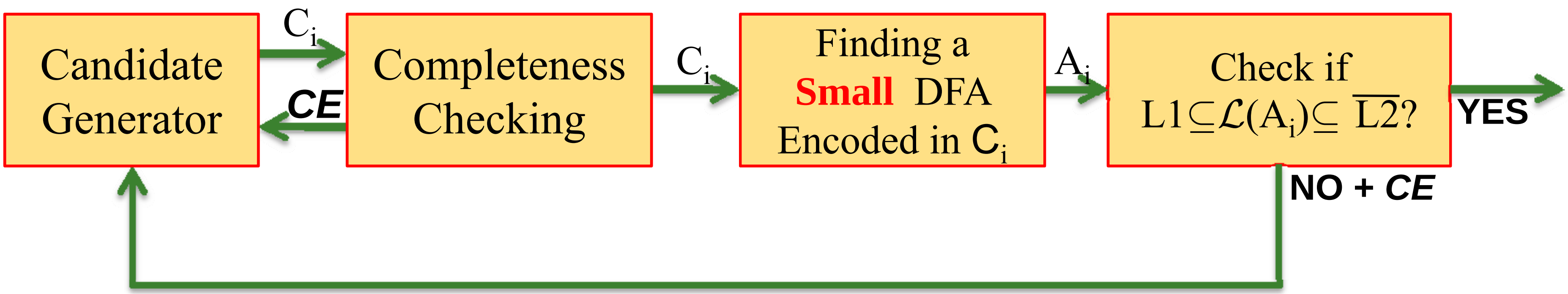


Adapt L^{Sep} for Compositional Verification

- Let L1 = M2 and $\overline{L2}$ = $P \cup \overline{M1}$, use L^{Sep} to find a separating DFA for L1 and L2.
- When $M2 \not\subseteq P \cup \overline{M1}$ (i.e., $M1 \cap M2 \not\subseteq P$), L^{Sep} can be modified to guarantee finding a string in M2, but not in $P \cup \overline{M1}$(i.e., $M1 \cap M2 \setminus P$).

Adapt L^{Sep} for Compositional Verification

- Use heuristics to find a small consistent DFA:



Candidate Generator

C_i

CE

Completeness Checking

C_i

Finding a **Small** DFA Encoded in C_i

A_i

Check if $L1 \subseteq \mathcal{L}(A_i) \subseteq \overline{L2}$?

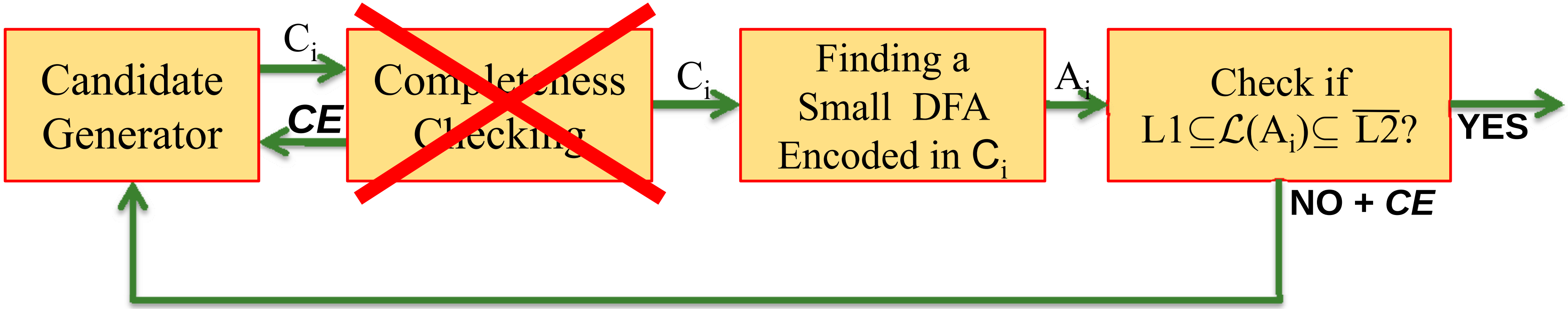
YES

NO + *CE*

Minimality is no longer guaranteed!

Adapt L^{Sep} for Compositional Verification

- Skip completeness checking:



Minimality is no longer guaranteed!