Satisfiability Solving and Tools

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Outline

1. Fundamental Concepts
2. Core algorithms of satisfiability problems
3. Heuristics
4. SAT competitions
5. Applications

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1. Fundamental Concepts
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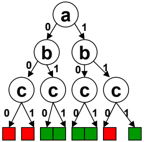
Boolean Satisfiability Problem(SAT Problem)

- Given a Boolean formula, find a assignment such that the function evaluates to 1, or prove that no such assignment exists (UNSAT).
  - EX. $F = (a \lor b) \land (\bar{a} \lor \bar{b} \lor c)$
    This function is SAT when $a = 1, b = 1, c = 1$
  - EX. $F = (a) \land (\bar{a} \lor b) \land (\bar{a} \lor \bar{b})$
    This function is *UNSAT*
- For n variables, there are 2^n possible truth assignments to be checked.



- First proved NP-Complete problem.
  - S. A. Cook, The complexity of theorem proving procedures, *Proceedings, Third Annual ACM Symp. on the Theory of Computing, 1971*.

Bolean Reasoning

- The central idea in Boolean reasoning, first given by Boole, is to reduce a given system of logical equations, and then to carry out the desired reasoning on that equation.
- e.g. Model checking $A \models f : L(A) \cap L(B_{\neg f}) = \emptyset$
- Fundamental tradeoff
  - canonical data structure (e.g. truth table, ROBDD)
    - data structure uniquely represents function
    - decision procedure is trivial(pointer comparison, DFS)
    - size of data structure is in general exponential

    item non-canonical data structure (e.g. AIG, CNF)
    - size of data structure is in general linear
    - systematic search for for satisfying assignment
    - decision may take an exponential amount of time

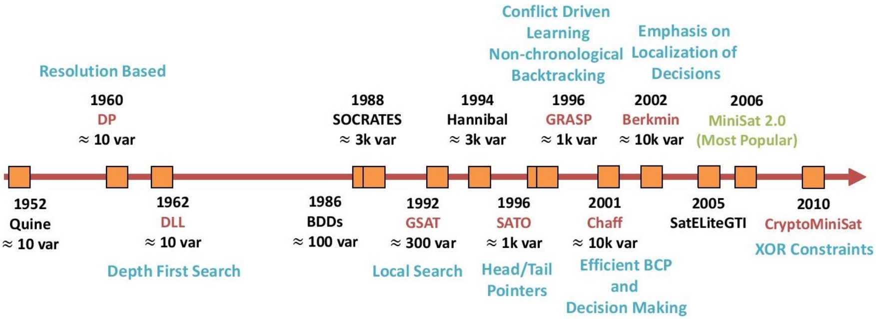
Boolean Satisfiability Solvers

- Boolean SAT solvers have been very successful recent years in the verification area, due to various nice heuristics
  - Support up to 10k variables, much more scalable than BDDs
  - Applications: equivalence checking and model checking
  - Applicable even for million-gate designs in EDA
- Popular SAT Solvers
  - MiniSat (2008 winner, the most popular one)
  - CryptoMiniSat (2011 winner)
  - glucose

Conjunctive Normal Form (CNF)

- A Boolean formula is represented as a CNF (i.e. Product of Sum).
- Linear structure for lots of variables and easy to add extra constraints
- Literal is a variable or its negation.
- CNF formula is a conjunction of clauses, where a clause is a disjunction of literals.
- For example:
  $(a \vee b \vee c) \wedge (\bar{a} \vee \bar{b} \vee c)$
  - Variable: a, b, c in this CNF formula.
  - Literals: $\bar{a}, \bar{b}, c$ are literals in $(\bar{a} \vee \bar{b} \vee c)$.
  - Clauses: $(a \vee b \vee c)$, $(\bar{a} \vee \bar{b} \vee c)$ are clauses in this CNF formula.

The Timeline of the SAT Solver



1952
Quine
≈ 10 var

Resolution Based
1960
DP
≈ 10 var

1962
DLL
≈ 10 var
Depth First Search

1986
BDDs
≈ 100 var

1988
SOCRATES
≈ 3k var

1992
GSAT
≈ 300 var
Local Search

1994
Hannibal
≈ 3k var

1996
SATO
≈ 1k var
Head/Tail Pointers

Conflict Driven Learning
Non-chronological Backtracking
1996
GRASP
≈ 1k var

2001
Chaff
≈ 10k var
Efficient BCP and Decision Making

Emphasis on Localization of Decisions
2002
Berkmin
≈ 10k var

2005
SatELiteGTI

2006
MiniSat 2.0
(Most Popular)

2010
CryptoMiniSat
XOR Constraints

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   - Davis-Putnam Algorithm
   - DPLL Algorithm
   - GRASP Algorithm
   - zChaff Algorithm
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CNF-Based SAT Algorithms

- Davis-Putnam (DP), 1960.
    - Explicit resolution based
    - May explode in memory
- Davis-Putnam-Logemann-Loveland (DPLL), 1962.
    - Search based
    - Most successful, basis for almost all modern SAT solvers
- GRASP, 1996
    - Conflict driven learning and non-chronological backtracking
- zChaff, 2001.
    - Efficient Boolean constraint propagation (BCP) algorithm (two watched literals)

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Davis-Putnam Algorithm

- M.Davis, H.Putnam, "A computing procedure for quantification theory" J. of ACM, 1960
- By repeating three satisfiability-preserving rules:
  - Unit propagation rule
  - Pure literal rule
  - Resolution rule
- eventually obtain:
  - $\bot \in F$ indicates UNSAT
  - $F = \top$ (a formula with no clauses indicates SAT)

DP Algorithm

DP Pseudo Code

```
Function DP($F$, $A$)
    forever
        if $\perp \in F$ then
        return $UNSAT$;
        if $F = \top$ then
        return $SAT$;

        $A \leftarrow Unit - Propagation(F, A)$;
        $A \leftarrow Pure - Literal(F, A)$;
        $A \leftarrow Resolution(F, A)$;
```

Unit Propagation Rule

- Suppose (a) is a unit clause, i.e. a clause contains only one literal.
  - Remove any instances of $\bar{a}$ from the formula.
  - Remove all clauses containing a.
- Example:
  - $(a) \wedge (\bar{a} \vee b \vee c) \wedge (a \vee \bar{b} \vee c) \wedge (\bar{a} \vee \bar{c} \vee d)$
    $\approx (b \vee c) \wedge (\bar{c} \vee d)$
  - $(a) \wedge (a \vee b) \approx$ *satisfiable*
  - $(a) \wedge (\bar{a}) \approx (\ )$ *unsatisfiable*

Pure Literal Rule

- If a literal appears only positively or only negatively, delete all clauses containing that literal.
- Example:
  $(\bar{a} \vee b \vee c) \wedge (\bar{a} \vee \bar{b} \vee c) \wedge (\bar{b} \vee c \vee d) \wedge (\bar{a} \vee \bar{c} \vee \bar{d})$
  $\approx (\bar{b} \vee c \vee d)$

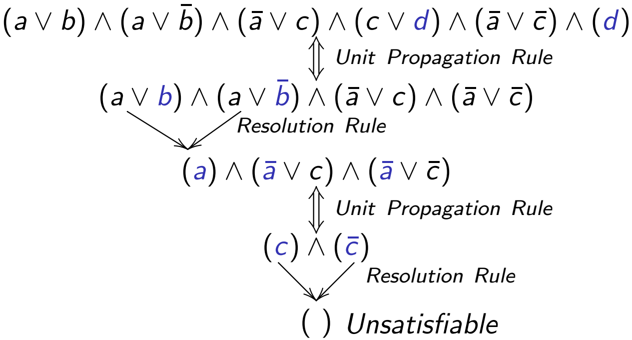
Resolution Rule

- For a single pair of clauses, $(a \vee l_1 \vee \cdots \vee l_m)$ and $(\bar{a} \vee k_1 \vee \cdots \vee k_n)$, resolution on a forms the new clause $(l_1 \vee \cdots \vee l_m \vee k_1 \vee \cdots \vee k_n)$.
- Example:
  $(a \vee b) \wedge (\bar{a} \vee c) \approx (b \vee c)$
  - If a is `True`, then for the formula to be `True`, c must be `True`.
  - If a is `False`, then for the formula to be `True`, b must be `True`.
  - So regardless of a, for the formula to be `True`, $b \vee c$ must be `True`.

Resolution Rule (cont.)

- Choose a propositional variable p which occurs positively in at least one clause and negatively in at least one other clause.
- Let P be the set of all clauses in which p occurs positively.
- Let N be the set of all clauses in which p occurs negatively.
- Replace the clauses in P and N with those obtained by resolving each clause in P with each clause in N.

Example 1



$$(a \vee b) \wedge (a \vee \bar{b}) \wedge (\bar{a} \vee c) \wedge (c \vee d) \wedge (\bar{a} \vee \bar{c}) \wedge (d)$$

$$\Updownarrow \; \textit{Unit Propagation Rule}$$

$$(a \vee b) \wedge (a \vee \bar{b}) \wedge (\bar{a} \vee c) \wedge (\bar{a} \vee \bar{c})$$

Resolution Rule

$$(a) \wedge (\bar{a} \vee c) \wedge (\bar{a} \vee \bar{c})$$

$$\Updownarrow \; \textit{Unit Propagation Rule}$$

$$(c) \wedge (\bar{c})$$

Resolution Rule

$$(\;)\; \textit{Unsatisfiable}$$

Example 2

- Solve $(a \vee b) \wedge (a \vee \bar{b}) \wedge (\bar{a} \vee c) \wedge (\bar{a} \vee \bar{c})$
- Wrong resolution:
  $(a \vee b) \wedge (a \vee \bar{b}) \wedge (\bar{a} \vee c) \wedge (\bar{a} \vee \bar{c})$ Use resolution rule
  $\approx (b \vee c) \wedge (\bar{b} \vee \bar{c})$ Use resolution rule
  $\approx (c \vee \bar{c})$ No rule can be used and no clause is empty!
  $\approx$ SAT → Wrong result!
- We have to resolve each clause in P with each clause in N.
- Correct resolution:
  - Choose a to do resolution
  - $P = \{(a \vee b), (a \vee \bar{b})\}$
  - $N = \{(\bar{a} \vee c), (\bar{a} \vee \bar{c})\}$
  - $R = \{(b \vee c), (b \vee \bar{c}), (\bar{b} \vee c), (\bar{b} \vee \bar{c})\}$
  - $(a \vee b) \wedge (a \vee \bar{b}) \wedge (\bar{a} \vee c) \wedge (\bar{a} \vee \bar{c})$
    $\approx (b \vee c) \wedge (b \vee \bar{c}) \wedge (\bar{b} \vee c) \wedge (\bar{b} \vee \bar{c})$ Replace P, N with R!
    $\approx$ ...
- Potential memory explosion problem (n → $n^2/4$)

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DPLL Algorithm

- M. Davis, G. Logemann and D. Loveland, "A Machine Program for Theorem-Proving", *Communications of ACM, 1962*. (New York Univ.)
- The basic framework for many modern SAT solvers.
- Main strategy
    - Decision Making
    - Unit Clause Rule
    - Implication
    - Conflict Detection
    - Backtracking

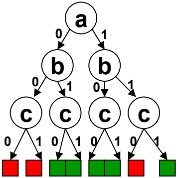
DPLL Algorithm

DPLL Pseudo Code

```
Function DPLL($F$, $A$)

    $A \leftarrow Unit - Propagation(F, A)$;
    if $A$ is inconsistent then
        return $UNSAT$;
    if $A$ assigns a value to every variable then
        return $SAT$;

    $v \leftarrow$ a variable not assigned a value by $A$;
    if DPLL($F$, $A \cup$ { $v =$ False }) $= SAT$
        return $SAT$;
    else
        return DPLL($F$, $A \cup$ { $v =$ True });
```



Boolean Constraint Propagation(a.k.a. Unit Propagation)

- Iteratively apply the unit clause rule until there is no unit clause available.
- Unit clause rule
  - A rule for elimination of one-literal clauses
  - An unsatisfied clause is a unit clause if it has exactly one unassigned literal.
  - The only unassigned literal, e.g. $\bar{c}$, is implied.
- Workhorse of DPLL based algorithms.

Basic DPLL Procedure - DFS

Caution: The graph on the right is drawn for the purpose of lecture. It is not seen in the algorithm implementation.

$(\bar{a} \vee b \vee c)$
$(a \vee c \vee d)$
$(a \vee c \vee \bar{d})$
$(a \vee \bar{c} \vee d)$
$(a \vee \bar{c} \vee \bar{d})$
$(\bar{b} \vee \bar{c} \vee d)$
$(\bar{a} \vee b \vee \bar{c})$
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Basic DPLL Procedure - DFS

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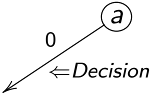

Basic DPLL Procedure - DFS

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$(a \vee c \vee d)$
$(a \vee c \vee \bar{d})$
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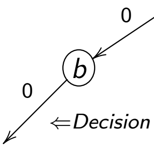

Basic DPLL Procedure - DFS

$(\bar{a} \vee b \vee c)$

$(a \vee c \vee d)$

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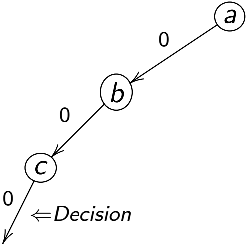

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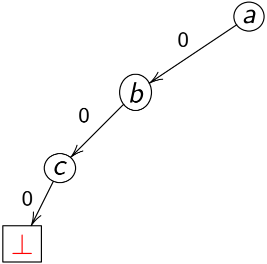
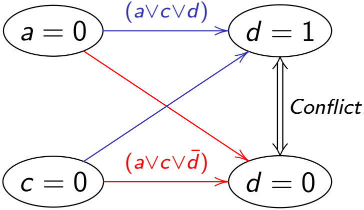


Implication Graph



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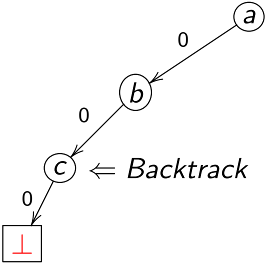

$\Leftarrow$ *Backtrack*

Basic DPLL Procedure - DFS

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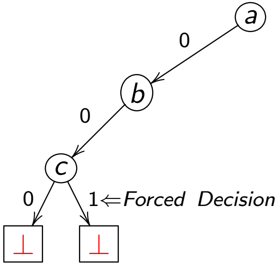

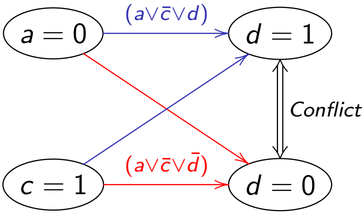

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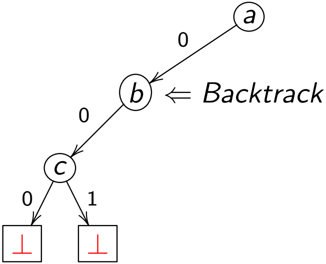

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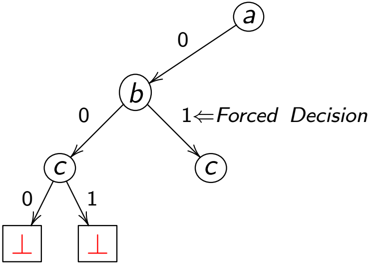

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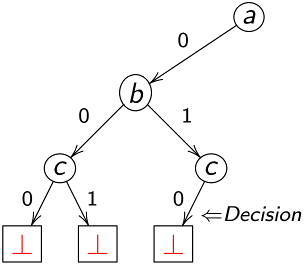

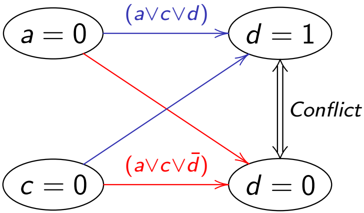

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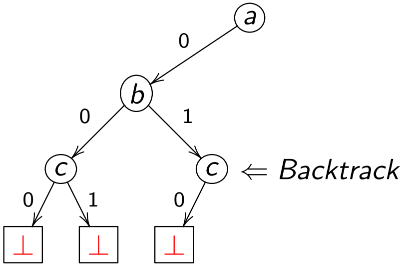

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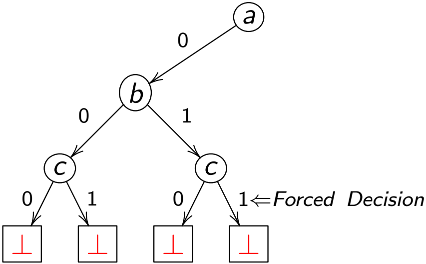

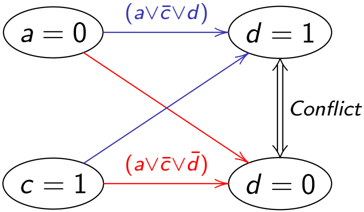

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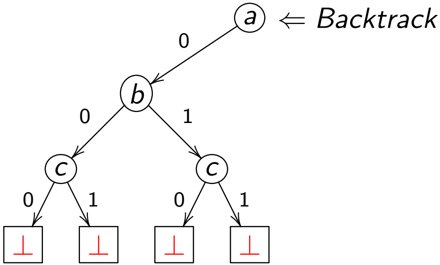

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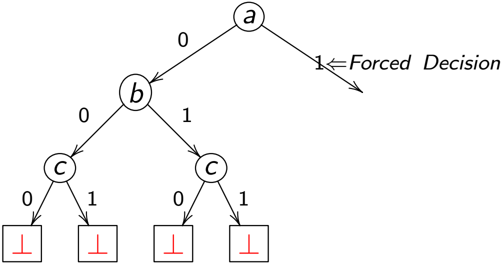

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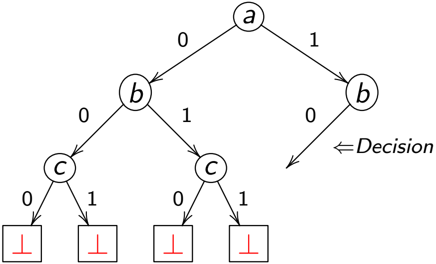

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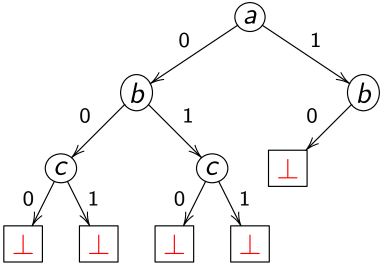

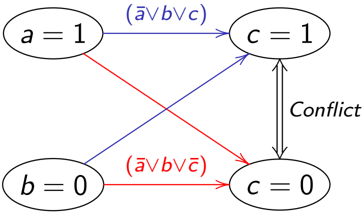

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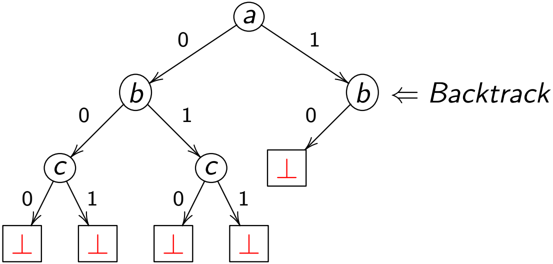
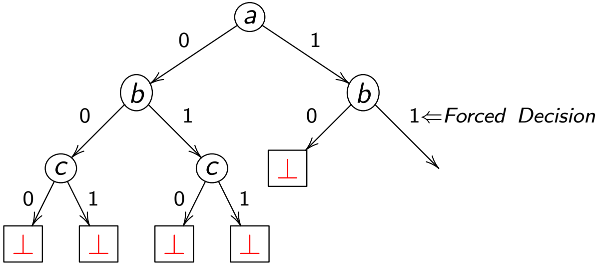
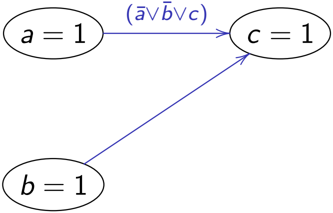
$(a \vee \bar{c} \vee d)$
$(a \vee \bar{c} \vee \bar{d})$
$(\bar{b} \vee \bar{c} \vee d)$
$(\bar{a} \vee b \vee \bar{c})$
$(\bar{a} \vee \bar{b} \vee c)$



Basic DPLL Procedure - DFS

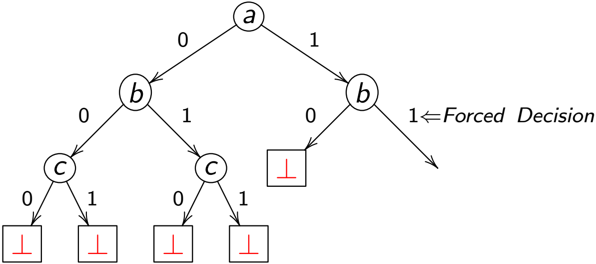
$(\bar{a} \vee b \vee c)$
$(a \vee c \vee d)$
$(a \vee c \vee \bar{d})$
$(a \vee \bar{c} \vee d)$
$(a \vee \bar{c} \vee \bar{d})$
$(\bar{b} \vee \bar{c} \vee d)$
$(\bar{a} \vee b \vee \bar{c})$
$(\bar{a} \vee \bar{b} \vee c)$

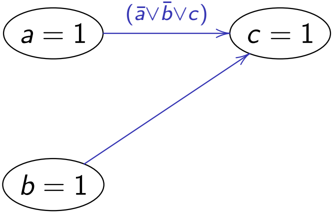




Basic DPLL Procedure - DFS

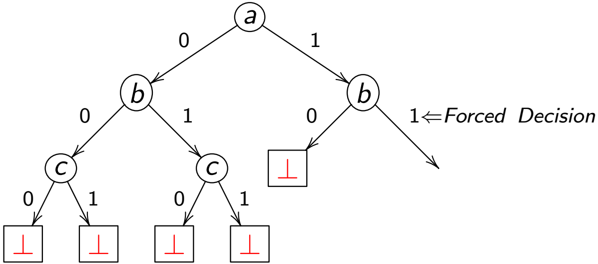
$(\bar{a} \vee b \vee c)$
$(a \vee c \vee d)$
$(a \vee c \vee \bar{d})$
$(a \vee \bar{c} \vee d)$
$(a \vee \bar{c} \vee \bar{d})$
$(\bar{b} \vee \bar{c} \vee d)$
$(\bar{a} \vee b \vee \bar{c})$
$(\bar{a} \vee \bar{b} \vee c)$

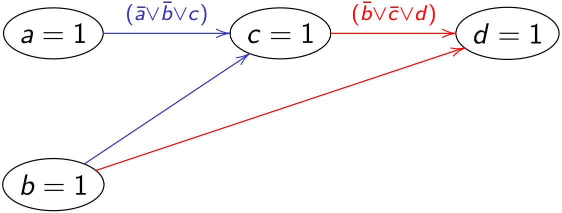




Basic DPLL Procedure - DFS

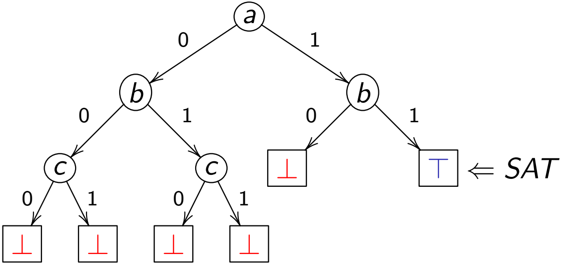
$(\bar{a} \vee b \vee c)$
$(a \vee c \vee d)$
$(a \vee c \vee \bar{d})$
$(a \vee \bar{c} \vee d)$
$(a \vee \bar{c} \vee \bar{d})$
$(\bar{b} \vee \bar{c} \vee d)$
$(\bar{a} \vee b \vee \bar{c})$
$(\bar{a} \vee \bar{b} \vee c)$

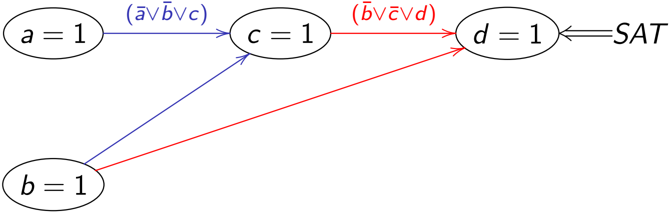




Basic DPLL Procedure - DFS

$(\bar{a} \vee b \vee c)$
$(a \vee c \vee d)$
$(a \vee c \vee \bar{d})$
$(a \vee \bar{c} \vee d)$
$(a \vee \bar{c} \vee \bar{d})$
$(\bar{b} \vee \bar{c} \vee d)$
$(\bar{a} \vee b \vee \bar{c})$
$(\bar{a} \vee \bar{b} \vee c)$





Features of DPLL

- Eliminate the potential memory explosion of DP
- Exponential time is still a problem
- Very limited size of problems are allowed
  - 32K word memory
  - Problem size limited by total size of clauses (about 1300 clauses)

Outline

GRASP

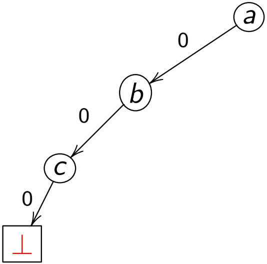
- Marques-Silva and Sakallah [SS96,SS99] (Univ. of Michigan)
  - J. P. Marques-Silva and K. A. Sakallah, "GRASP – A New Search Algorithm for Satisfiability", *Proc.ICCAD, 1996*.
  - J. P. Marques-Silva and Karem A. Sakallah, "GRASP: A Search Algorithm for Propositional Satisfiability", *IEEE Trans. Computers, 1999*.
- Incorporate conflict driven learning and non-chronological backtracking.
- Practical SAT problem instances can be solved in reasonable time.

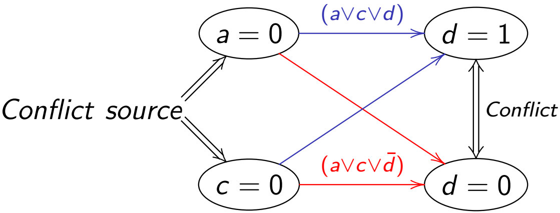
SAT Improvements

- Conflict driven learning
  - Once we encounter a conflict, figure out the cause(s) of this conflict and prevent to see this conflict again.
  - Add learned clause (conflict clause) which is the negative proposition of the conflict source.
- Non-chronological backtracking
  - After getting a learned clause from the conflict analysis, we backtrack to the "next-to-the-last" variable in the learned clause.
  - Instead of backtracking one decision at a time.

Conflict Driven Learning

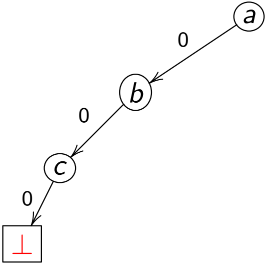
$(\bar{a} \vee b \vee c)$
$(a \vee c \vee d)$
$(a \vee c \vee \bar{d})$
$(a \vee \bar{c} \vee d)$
$(a \vee \bar{c} \vee \bar{d})$
$(\bar{b} \vee \bar{c} \vee d)$
$(\bar{a} \vee b \vee \bar{c})$
$(\bar{a} \vee \bar{b} \vee c)$

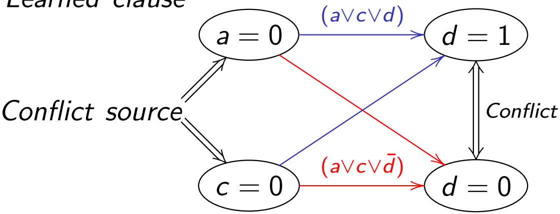




Conflict Driven Learning

$(\bar{a} \vee b \vee c)$
$(a \vee c \vee d)$
$(a \vee c \vee \bar{d})$
$(a \vee \bar{c} \vee d)$
$(a \vee \bar{c} \vee \bar{d})$
$(\bar{b} \vee \bar{c} \vee d)$
$(\bar{a} \vee b \vee \bar{c})$
$(\bar{a} \vee \bar{b} \vee c)$
$(a \vee c)$ *Learned clause*





Non-Chronological Backtracking

$(\bar{a} \vee b \vee c)$

$(a \vee c \vee d)$

$(a \vee c \vee \bar{d})$

$(a \vee \bar{c} \vee d)$

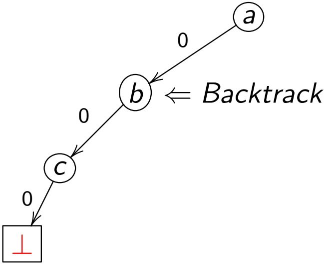
$(a \vee \bar{c} \vee \bar{d})$

$(\bar{b} \vee \bar{c} \vee d)$

$(\bar{a} \vee b \vee \bar{c})$

$(\bar{a} \vee \bar{b} \vee c)$

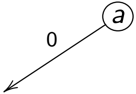
$(a \vee c)$ *Learned clause*

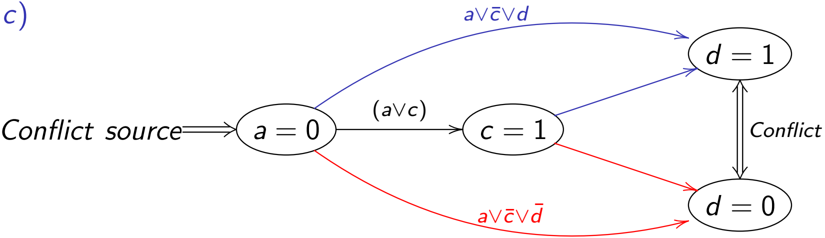


- ‘a’ is the next-to-the-last variable in the (current) learned clause.
  - c is the last (assigned) variable in this learned clause so a is called the next-to-the-last variable
  - Because of this learned clause, when a is assigned 0 then c will be implied and we don’t have to make decision for c
- After doing non-chronological backtracking, we will not forgive the path $a = 0, b = 0 \ldots$ if needed.

Non-Chronological Backtracking

$(\bar{a} \vee b \vee c)$
$(a \vee c \vee d)$
$(a \vee c \vee \bar{d})$
$(a \vee \bar{c} \vee d)$
$(a \vee \bar{c} \vee \bar{d})$
$(\bar{b} \vee \bar{c} \vee d)$
$(\bar{a} \vee b \vee \bar{c})$
$(\bar{a} \vee \bar{b} \vee c)$
$(a \vee c)$





Non-Chronological Backtracking

$(\bar{a} \lor b \lor c)$
$(a \lor c \lor d)$
$(a \lor c \lor \bar{d})$
$(a \lor \bar{c} \lor d)$
$(a \lor \bar{c} \lor \bar{d})$
$(\bar{b} \lor \bar{c} \lor d)$
$(\bar{a} \lor b \lor \bar{c})$
$(\bar{a} \lor \bar{b} \lor c)$
$(a \lor c)$
(a) *Learned clause*

- Since there is only one variable in the learned clause, no one is the next-to-the-last variable.
- Backtrack all decisions

Non-Chronological Backtracking

$(\bar{a} \vee b \vee c)$

$(a \vee c \vee d)$

$(a \vee c \vee \bar{d})$

$(a \vee \bar{c} \vee d)$

$(a \vee \bar{c} \vee \bar{d})$

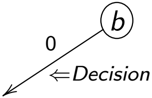
$(\bar{b} \vee \bar{c} \vee d)$

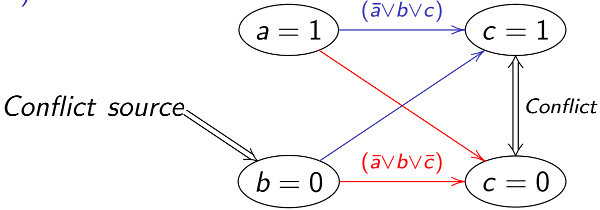
$(\bar{a} \vee b \vee \bar{c})$

$(\bar{a} \vee \bar{b} \vee c)$

$(a \vee c)$

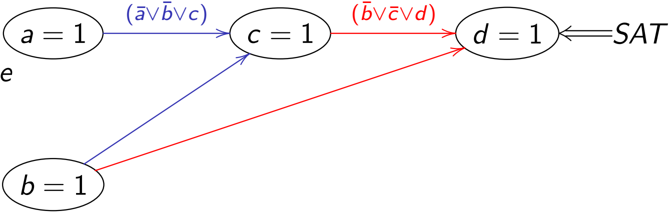
(a)





Non-Chronological Backtracking

$(\bar{a} \vee b \vee c)$
$(a \vee c \vee d)$
$(a \vee c \vee \bar{d})$
$(a \vee \bar{c} \vee d)$
$(a \vee \bar{c} \vee \bar{d})$
$(\bar{b} \vee \bar{c} \vee d)$
$(\bar{a} \vee b \vee \bar{c})$
$(\bar{a} \vee \bar{b} \vee c)$
$(a \vee c)$
(a)
(b) *Learned clause*



More on Implication Graph

- How to determine the conflict source?

$(\bar{a} \lor b \lor c)$

$(a \lor c \lor d)$

$(a \lor c \lor \bar{d})$

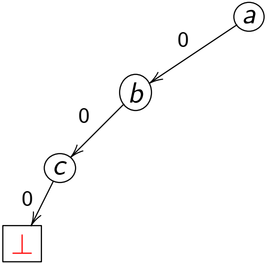
$(a \lor \bar{c} \lor d)$

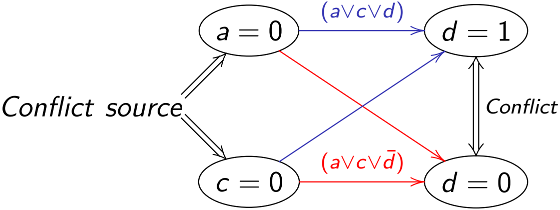
$(a \lor \bar{c} \lor \bar{d})$

$(\bar{b} \lor \bar{c} \lor d)$

$(\bar{a} \lor b \lor \bar{c})$

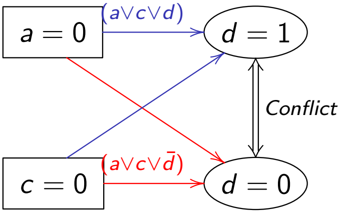
$(\bar{a} \lor \bar{b} \lor c)$





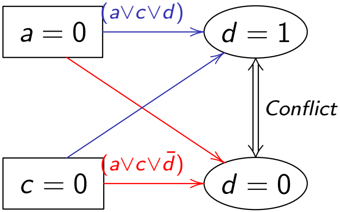
More on Implication Graph (cont.)

- How to determine the conflict source?
- We need to find a Cut on the Implication Graph, such that every path from the decision nodes to the conflict nodes must pass through it.
  - Decision nodes are the variables assigned to value in each decision process
  - Implication nodes are the variables assigned to value by implication
  - Conflict nodes are where the conflict shows up



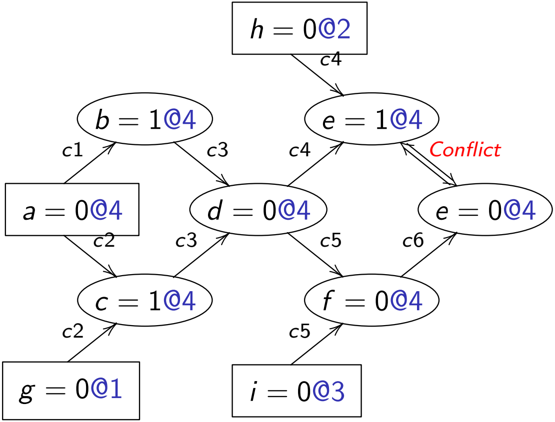
More on Implication Graph(cont.)

- Take all decision nodes to form a cut (technique used by *Rel_Sat*, [R. Bayardo R. Shrag, 1997])
- In this case, the cut consists of a=0 c=0.



Unique Implication Point(UIP)

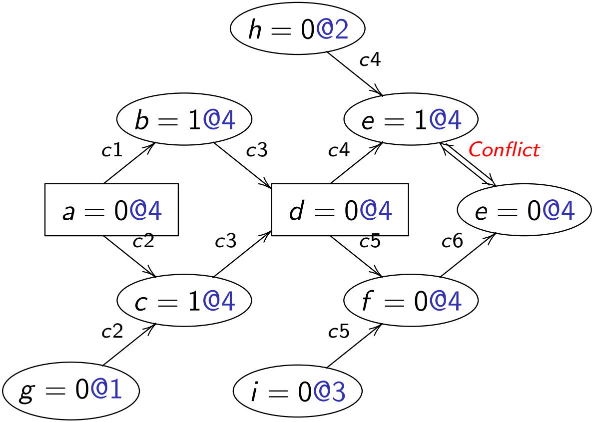
- Taking all decision nodes to form a cut may result in less valuable learnt clause.
- How about learning a clause that is close related to the conflict?



$$(a \vee b)$$
$$(a \vee c \vee g)$$
$$(\bar{b} \vee \bar{c} \vee d)$$
$$(\bar{d} \vee e \vee h)$$
$$(\bar{d} \vee f \vee i)$$
$$(\bar{e} \vee \bar{f})$$
$$(\bar{d} \vee h \vee i)$$

Unique Implication Point(UIP)

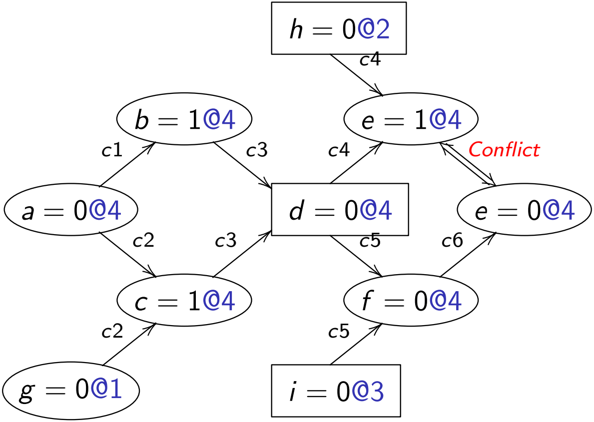
- A UIP is any node at the current decision level (@4) such that any path from decision variable (a=0@4) to the conflict nodes must pass through it.



$$(a \vee b)$$
$$(a \vee c \vee g)$$
$$(\bar{b} \vee \bar{c} \vee d)$$
$$(\bar{d} \vee e \vee h)$$
$$(\bar{d} \vee f \vee i)$$
$$(\bar{e} \vee \bar{f})$$
$$(\bar{d} \vee h \vee i)$$

Unique Implication Point(UIP)

- First-UIP Learning Scheme: used by MiniSAT and zChaff
- Set the cut right before the first UIP is encountered on the path leading from the conflict nodes.



$(a \vee b)$

$(a \vee c \vee g)$

$(\bar{b} \vee \bar{c} \vee d)$

$(\bar{d} \vee e \vee h)$

$(\bar{d} \vee f \vee i)$

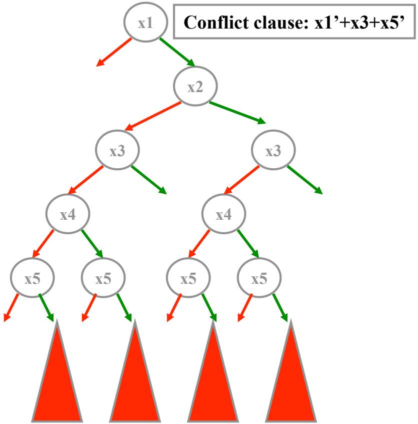
$(\bar{e} \vee \bar{f})$

$(\bar{d} \vee h \vee i)$

$(h \vee d \vee i)$

What's the big deal?

- We can know learn the related clause to each conflict we encountered.
- Significantly prune the search space because learned clause is useful forever!



Search Completeness

- With conflict driven learning, SAT search is still guaranteed to be complete.
- SAT search becomes a decision stack instead of a binary decision tree.
- When encountering a conflict, the conflict analysis does the following tasks:
    - Learned clause
    - Indicate where to backtrack
    - Learned implication

SAT Becomes Practical

- Conflict driven learning greatly increases the capacity of SAT solvers (several thousand variables) for structured problems.
- Realistic applications became plausible.
  - Usually thousands and even millions of variables
  - Typical EDA applications can make use of SAT including circuit verification, FPGA routing and many other applications
- Research direction changes towards more efficient implementations.

Outline

1. Fundamental Concepts

2. Core algorithms of satisfiability problems
   - Davis-Putnam Algorithm
   - DPLL Algorithm
   - GRASP Algorithm
   - zChaff Algorithm

3. Heuristics

4. SAT competitions

5. Applications

zChaff

- M. Moskewicz, C. Madigan, Y. Zhao, L. Zhang, S. Malik,"Chaff: Engineering an Efficient SAT Solver" *Proc. DAC 2001*. (UC Berkeley, MIT and Princeton Univ.)
- Make the core operations fast.
  - After profiling, the most time-consuming parts are Boolean Constraint Propagation (BCP) and Decision.
- As always, pruning search space (i.e. conflict driven learning) is important.

BCP Algorithm

- When can BCP (Unit propagation, implication) occur ?
  - All literals but one are assigned to False in a clause.

$$\textit{The implied cases of } (v1 \vee v2 \vee v3):$$
$$(0 \vee 0 \vee v3) \textit{ or } (0 \vee v2 \vee 0) \textit{ or } (v1 \vee 0 \vee 0)$$

  - For an N-literal clause, this can only occur after $N-1$ literals have been assigned to `False`.
  - So, (theoretically) we could completely ignore the first $N-2$ assignments to this clause.
  - Two watched Literals:
    In reality, we pick two literals in each clause to "watch" and thus can ignore any assignments to the other literals in the clause.
  - This is not a pruning technique, but saves time while performing BCP.

BCP Algorithm

- Heuristically start with watching two unassigned literals in each clause.
- When one of the two watched literals is assigned `True`, this clause becomes `True`.
- When one of the two watched literals is assigned `False`, we send the clause into an Update-Watch queue to do one of the followings:
  - 1. Updating (there exists another unassigned literal)
  - 2. BCP (only one watched literal unassigned)
  - 3. Conflict handling (all literals are `False`)

BCP Algorithm

- Initially, pick any two literals in each clause as the watched literals.
  - Green: watched literals
- Clauses with only one literal are detected at the mean time.

$v2 \lor v3 \lor v1 \lor v4 \lor v5$

$v1 \lor v2 \lor \overline{v3}$

$v1 \lor \overline{v2}$

$\overline{v1} \lor v4$

$\overline{v1} \longleftarrow$ Detect unit clause

BCP Algorithm

- We begin by processing the assignment $v1 = F$
  - Implied by the unit clause $\overline{v1}$

$$v2 \vee v3 \vee v1 \vee v4 \vee v5$$
$$v1 \vee v2 \vee \overline{v3}$$
$$v1 \vee \overline{v2}$$
$$\overline{v1} \vee v4$$

$State : v1 = F$

$Pending :$

BCP Algorithm

- Need not process clauses where watched literals are set to `True`.
  - Because those clauses are now satisfied.

$$\begin{array}{ll} & v2 \lor v3 \lor v1 \lor v4 \lor v5 \\ & v1 \lor v2 \lor \overline{v3} \\ & v1 \lor \overline{v2} \\ \Rightarrow & \overline{v1} \lor v4 \end{array}$$

$$State : v1 = F$$

$$Pending :$$

BCP Algorithm

- Need not process clauses where neither watched literal is assigned.
  - Because those clause are definitely not a unit clause.

$$\Rightarrow \quad v2 \vee v3 \vee v1 \vee v4 \vee v5$$

$$v1 \vee v2 \vee \overline{v3}$$

$$v1 \vee \overline{v2}$$

$$\overline{v1} \vee v4$$

$$State : v1 = F$$

$$Pending :$$

BCP Algorithm

- Only examine clauses where a watched literal is set to `False` due to the assignment.

$$
\begin{array}{ll}
 & v2 \lor v3 \lor v1 \lor v4 \lor v5 \\
\Rightarrow & v1 \lor v2 \lor \overline{v3} \\
\Rightarrow & v1 \lor \overline{v2} \\
 & \overline{v1} \lor v4
\end{array}
$$

$State : v1 = F$

$Pending :$

BCP Algorithm

- For the second clause, we replace $v1$ with $\overline{v3}$ as a new watched literal because $\overline{v3}$ is not assigned to `False`.

$$\Rightarrow \quad \begin{array}{l} v2 \lor v3 \lor v1 \lor v4 \lor v5 \\ v1 \lor v2 \lor \overline{v3} \\ v1 \lor \overline{v2} \\ \overline{v1} \lor v4 \end{array} \qquad \Longrightarrow \quad \begin{array}{l} v2 \lor v3 \lor v1 \lor v4 \lor v5 \\ v1 \lor v2 \lor \overline{v3} \\ v1 \lor \overline{v2} \\ \overline{v1} \lor v4 \end{array}$$

$State : v1 = F$ $\qquad$ $State : v1 = F$

$Pending :$ $\qquad$ $Pending :$

BCP Algorithm

- The third clause is a unit clause.
- We record the new implication of $\overline{v2}$, and add it to the queue of assignments to process.

$$\Rightarrow \quad \begin{array}{l} v2 \vee v3 \vee v1 \vee v4 \vee v5 \\ v1 \vee v2 \vee \overline{v3} \\ v1 \vee \overline{v2} \\ \overline{v1} \vee v4 \end{array} \qquad \begin{array}{l} v2 \vee v3 \vee v1 \vee v4 \vee v5 \\ v1 \vee v2 \vee \overline{v3} \\ v1 \vee \overline{v2} \\ \overline{v1} \vee v4 \end{array}$$

$$State : v1 = F \qquad\qquad State : v1 = F$$

$$Pending : \qquad \Longrightarrow \qquad Pending : (v2 = F)$$

BCP Algorithm

- Next, for $\overline{v2}$, only the first two clauses are examined.
  - For the first clause, replace $v2$ with $v4$ as a new watched literal.

$$
\begin{array}{llcl}
\Rightarrow & v2 \vee v3 \vee v1 \vee v4 \vee v5 & \Longrightarrow & v2 \vee v3 \vee v1 \vee v4 \vee v5 \\
\Rightarrow & v1 \vee v2 \vee \overline{v3} & & v1 \vee v2 \vee \overline{v3} \\
 & v1 \vee \overline{v2} & & v1 \vee \overline{v2} \\
 & \overline{v1} \vee v4 & & \overline{v1} \vee v4 \\
 & & & \\
 & State : v1 = F, v2 = F & & State : v1 = F, v2 = F \\
 & & & \\
 & Pending : & \Longrightarrow & Pending : (v3 = F)
\end{array}
$$

BCP Algorithm

- Next, for $\overline{v3}$, only the first clause is examined.
  - For the first clause, replace $v3$ with $v5$ as a new watched literal.
  - Since there are no pending assignments, and no conflict, BCP terminates and we make a decision. Both $v4$ and $v5$ are unassigned. Let's say we assign $v4 =$ `True` and proceed.

$$\Rightarrow \quad \begin{array}{l} v2 \vee v3 \vee v1 \vee v4 \vee v5 \\ v1 \vee v2 \vee \overline{v3} \\ v1 \vee \overline{v2} \\ \overline{v1} \vee v4 \end{array} \quad \Longrightarrow \quad \begin{array}{l} v2 \vee v3 \vee v1 \vee v4 \vee v5 \\ v1 \vee v2 \vee \overline{v3} \\ v1 \vee \overline{v2} \\ \overline{v1} \vee v4 \end{array}$$

$$\begin{array}{ll} State: v1 = F, v2 = F, & \quad State: v1 = F, v2 = F, \\ \qquad v3 = F & \quad \qquad v3 = F \\ Pending: & \quad Pending: \end{array}$$

BCP Algorithm

- Next, for $v4$, all clauses are satisfied.
- Depend on implementation, it may continue and assign value to $v5$.
- The instance is SAT, and we are done.

$$v2 \vee v3 \vee v1 \vee v4 \vee v5$$
$$v1 \vee v2 \vee \overline{v3}$$
$$v1 \vee \overline{v2}$$
$$\overline{v1} \vee v4$$

$$\begin{aligned} State &: v1 = F, v2 = F, \\ &\quad v3 = F, v4 = T \\ Pending &: \end{aligned}$$

BCP Algorithm Summary

- During forward progress: Decisions and Implications
  - Only need to examine clauses where watched literal is set to F
  - Can ignore any assignments of literals to T
  - Can ignore any assignments of non-watched literals
- During backtrack: Unwind Assignment Stack
  - No action is required at all to unassigned variables
  - But it is computation-intensive part in SATO (*SATO: an Efficient Propositional Prover. Hantao Zhang*. Department of Computer Science. The University of Iowa. Iowa City, IA 52242-1419, USA*)
- Overall minimize clause access

Outline

1. Fundamental Concepts

2. Core algorithms of satisfiability problems

3. Heuristics
   - Decision heuristics
   - Restart mechanism

4. SAT competitions

5. Applications

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Make Decision

- Because we want to prove that the target Boolean formula is satisfiable or not, we should start with guessing the state (`True` or `False`) of a variable until the proof is done.
- Some strategy:
  - Random
  - Dynamic Largest Individual Sum (DLIS)
  - Variable State Independent Decaying Sum (VSIDS)

RAND and DLIS

- Random
  - Simply select an unassigned variable and a value randomly for the next decision.
- Dynamic Largest Individual Sum (DLIS)
  - At each decision simply choose the assignment that satisfies the most unsatisfied clauses.
  - Simple and intuitive.
  - However, considerable work is required to maintain the statistics.
  - The total effort required is much more than the effort for the BCP algorithm in zChaff.

VSIDS

- Variable State Independent Decaying Sum (VSIDS)
  - Each variable in each polarity has a counter which is initialized to zero.
  - When a new clause is added to the database, the counter associated with each literal in this clause is incremented.
  - The (unassigned) variable and polarity with the highest counter is chosen at each decision.
  - Ties are broken randomly by default configuration.
  - Periodically, all the counters are divided by a constant.

VSIDS (cont.)

- VSIDS attempts to satisfy the conflict clauses but particularly attempts to satisfy recent learned clauses.
- Difficult problems generate many conflicts (and therefore many conflict clauses), the conflict clauses dominate the problem in terms of literal count.
- Since it is independent of the variable state, it has very low overhead.
- The average rum time overhead in zChaff:
  - BCP: about 80%
  - Decision: about 10%
  - Conflict analysis: about 10%

BerkMin

- E. Goldberg, and Y. Novikov, "BerkMin: A Fast and Robust Sat-Solver", *Proc. DATE 2002*. (Cadence Berkeley Labs and Academy of Sciences in Belarus)
- BerkMin tries to satisfy the most recent clause.
- The clause database is organized as a stack.
- The clauses of the original Boolean formula are located at the bottom of the stack and each new conflict clause is added to the top of the stack.
- The current top clause is the an unsatisfied clause which is the closest to the top of the stack.
- When making decision, choose the most active unassigned variable in the current top clause by using VSIDS.

Outline

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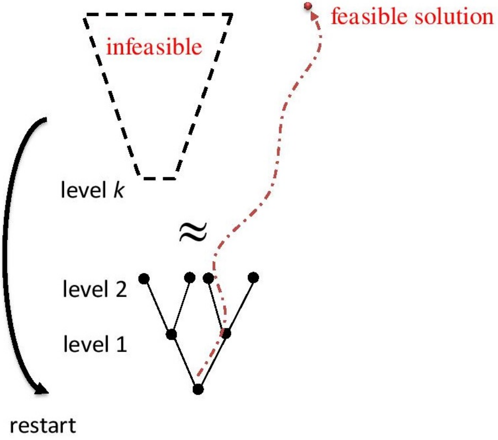
Restart Motivation

- Best time to restart:
  when algorithm spends too much time under a wrong branch



Restart

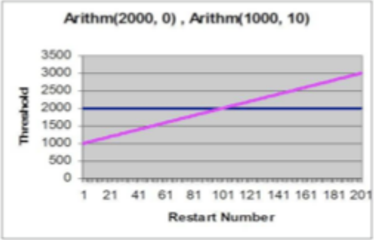
- Motivation: avoid spending too much time in “bad” branches.
  - no easy-to-find satisfying assignment
  - no opportunity for fast learning of strong clauses.
- All modern SAT solvers use a restart policy.
  - Following various criteria, the solver is forced to backtrack to level 0.
  - Abandon the current search tree and reconstruct a new one.
  - The clauses learned prior to the restart are still there after the restart and can help pruning the search space.
- Restarts have crucial impact on performance.
  - Reduce variance - increase robustness in the solver.

The Basic Measure for Restarts

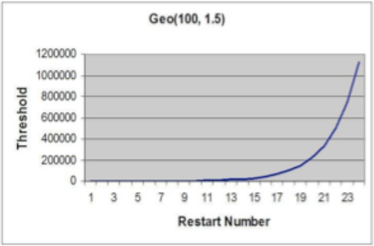
- All existing techniques use the number of conflicts learned as of the previous restart.
- The difference is only in the method of calculating the threshold.

Restarts strategies

- Arithmetic (or fixed) series
  - Used in Berkmin, Eureka, zChaff, Siege

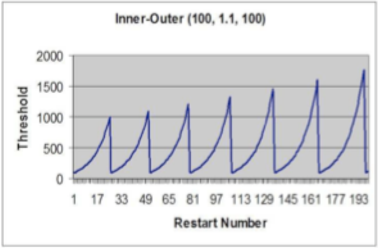


- Geometric series
  - Used in Minisat 2007



Restarts strategies

- Inner-Outer Geometric series
  - Used in Picosat



Other Issues

- Incremental SAT
  - Take apart the clause database.
    - Solve the first part and record the learned information.
    - If it is UNSAT, then stop.
    - If it is SAT, then add the next part to solve.
    - And so on...
  - Add Constraints according to the previous results
    - Since relevant learnt clauses are preserved, we speed up the later exploration.

Other Issues

- Reducing Learnt Clause
  - Large CNF can result in large amount of learnt clauses, and some of them may not be useful until later
  - They slow down BCP
- Remove learnt clauses periodically
  - Keep a certain number of learnt clauses
  - Minisat removes half of the learnt clauses if the number of clauses reaches threshold (which grows geometrically)
  - Glucose keeps short learnt clauses forever, but removes long ones if if the number of clauses reaches threshold (which grows arithmetically)

Other Issues

- Refutation proof, i.e., proof of UNSAT (Ex.Resolution Proof)
- Parallel computation
- Memory management
- etc...

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1. Fundamental Concepts
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4. **SAT competitions**
5. Applications

SAT competitions

- From March to June
- The international SAT Competitions (Starting from 2002)
  http://www.satcompetition.org/
  - Three main categories of benchmarks:
    Application(Industrial), Hard Combinatorial(Crafted), Random
  - Three Evaluation in each category:
    SAT, UNSAT, ALL(SAT + UNSAT)
  - Separate sequential and parallel since 2011
- SAT-Race (**2015**, 2010, 2008, 2006)
  http://baldur.iti.kit.edu/sat-race-2015/
- SAT Challenge 2012
  http://baldur.iti.kit.edu/SAT-Challenge-2012/

Famous SAT Solvers

- MiniSat, http://minisat.se/
  - Silver in 2005, Gold in 2006 and 2008
  - Well-known for its compact and simple implementation
  - Originally only 600 lines of C code in total
    but contains most algorithms mentioned in the slide!!
  - A category since 2009 called Minisat Hack
- SATzilla, http://www.cs.ubc.ca/labs/beta/Projects/SATzilla/
  - Gold in 2007, 2009, and 2012
  - Evaluate the problem instance first
  - Select an appropriate solver to solve

Famous SAT Solvers

- ppfolio, http://www.cril.univ-artois.fr/˜roussel/ppfolio/
    - Win a total of 16 medals in 2011
    - Assign cores to the five solvers in use.
- Winners of recent years
    - glucose, http://www.labri.fr/perso/lsimon/glucose/
    - Lingeling, http://fmv.jku.at/lingeling

Outline

The Usage of the MiniSat (Static Build)

- MiniSat Page: http://minisat.se/
- The newest version: 2.2.0
- Use MiniSat to find a solution of $F = (x_0 \vee x_1 \vee x_2) \wedge (\overline{x_1} \vee x_2)$.
  - Go to MiniSat Page to download it.
  - Tar the .gz file `tar -zxvf minisat-2.2.0.tar.gz`
  - Change to directory "core" `cd core`
  - Modify path `export MROOT=../`
  - Make and compile in directory "core" `make`
  - Build DIMACS CNF file for problem you want to solve *http://www.satcompetition.org/2009/format-benchmarks2009.html*
  - Run the minisat to solve problem `./minisat CnfFileName ResultFileName`

DIMACS CNF Format

- It is a standard format for the input files (CNF files) of SAT solvers.
  - Use c to write comments
  - Start with p cnf VarialbeNumber ClauseNumber
  - Write the clause with integer(with/without "-") for representing the literals
  - Use "0" to mark the end of a clause
- Example: $(x_0 \vee x_1 \vee x_2) \wedge (\overline{x_1} \vee x_2)$

  ```
  c this is a simple DIMACS cnf, use 1, 2, 3 for x0, x1, x2 respectively
  p cnf 3 2
  1 2 3 0
  -2 3 0
  ```
- What if we want yet another solution?
  Add block clauses and solve again!
  Problem: starting from scratch, file I/O

The Usage of the MiniSat (C++ API)

- MiniSat Page: http://minisat.se/
- MiniSat fork with CMake Integration: https://github.com/master-keying/minisat/
- MiniSat provides elegant c++ API

The Usage of the MiniSat (C++ API)

- Usage
  - Unzip the package `unzip minisat-master.zip`
  - Build minisat to target directory (If you would like a static build of MiniSat, do not build under the recent directory because of naming alias) `cmake -S . -B path`
  - Make and compile minisat build `make`
  - Write your code(details in later slides) `vim main.cpp`
  - Provide CMakeLists.txt(details in later slides vim CMakeLists.txt
  - Make and compile your program `cmake; make; ./demo`

API Usage

- Solver.h class Solver
  - newVar(bool polarity, bool dvar)
  - addClause(vec$<$ *Lit* $>$ ps)
  - addClause(Lit p)
  - addEmptyClause()
  - simplify()
  - solve()
- Your program
  - Construct Solver Solver s;
  - Add all constraints(clauses) s.addClause(ps)
  - solve s.solve();

CMake Guide

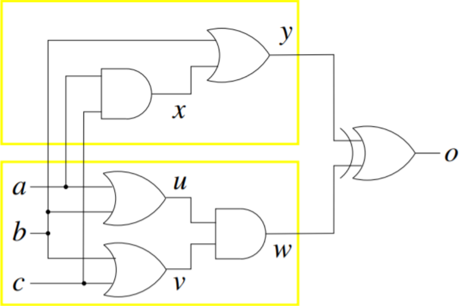
- Trivial CMakeList.txt
  - Identify CMake version
    `cmake_minimum_required(VERSION 3.5)`
  - Project name
    `project (projectname LANGUAGES CXX)`
  - Include all codes
    `add_executable(exename main.cpp)`
  - Add subdirectory
    `add_subdirectory(dir)`
  - Link to libminisat.a
    `target_link_libraries(exename MiniSat::libminisat)`
- Of course, you can build on your own, but the previous five lines is enough for now.

Demo 1 - Simple case

- Solve $(A \vee B \vee C) \wedge (\bar{A} \vee B \vee C) \wedge (A \vee \bar{B} \vee C) \wedge (A \vee B \vee \bar{C})$
- What if we need more than one solution? Add code!

Demo 2 - Equivalence Checking

- Check if the two circuits in the yellow boxes are equivalent?



$$\begin{aligned} & {\color{blue}o} \wedge \\ (x &\leftrightarrow a \wedge c) \wedge \\ (y &\leftrightarrow b \vee x) \wedge \\ (u &\leftrightarrow a \vee b) \wedge \\ (v &\leftrightarrow b \vee c) \wedge \\ (w &\leftrightarrow u \wedge v) \wedge \\ (o &\leftrightarrow y \oplus w) \end{aligned}$$

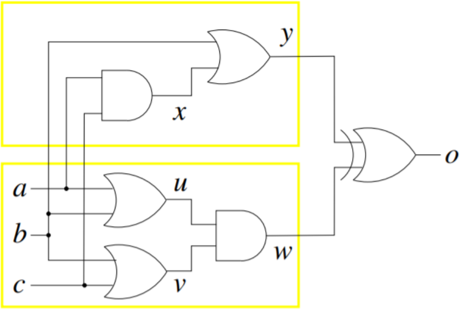
How to write CNF for circuits?

- Tseytin transformation
- $A \rightarrow B$ $(\bar{A} \vee B)$
- $A \leftrightarrow B$ $(\bar{A} \vee B) \wedge (A \vee \bar{B})$

Type	Operation	CNF Sub-expression
AND	$C = A \cdot B$	$(\bar{A} \vee \bar{B} \vee C) \wedge (A \vee \bar{C}) \wedge (B \vee \bar{C})$
NAND	$C = \overline{A \cdot B}$	$(\bar{A} \vee \bar{B} \vee \bar{C}) \wedge (A \vee C) \wedge (B \vee C)$
OR	$C = A + B$	$(A \vee B \vee \bar{C}) \wedge (\bar{A} \vee C) \wedge (\bar{B} \vee C)$
NOR	$C = \overline{A + B}$	$(A \vee B \vee C) \wedge (\bar{A} \vee \bar{C}) \wedge (\bar{B} \vee \bar{C})$
NOT	$C = \bar{A}$	$(\bar{A} \vee \bar{C}) \wedge (A \vee C)$
XOR	$C = A \oplus B$	$(\bar{A} \vee \bar{B} \vee \bar{C}) \wedge (A \vee B \vee \bar{C}) \wedge (A \vee \bar{B} \vee C) \wedge (\bar{A} \vee B \vee C)$
XNOR	$C = \overline{A \oplus B}$	$(\bar{A} \vee \bar{B} \vee C) \wedge (A \vee B \vee C) \wedge (A \vee \bar{B} \vee \bar{C}) \wedge (\bar{A} \vee B \vee \bar{C})$

Equivalence Checking

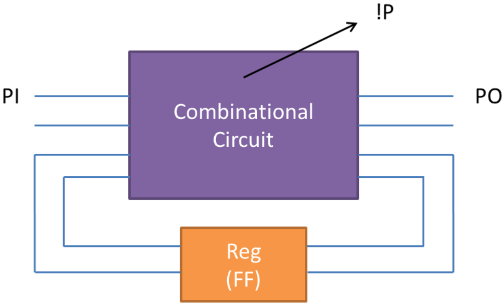
- Miter: Link the output of the two circuits with an XOR
- If the circuits are equivalent, signal O should always be False.
- By asserting O as a unit clause, the CNF formula should be UNSAT if the circuits are equivalent.



$$\begin{array}{r}
o \;\wedge \\
(x \leftrightarrow a \wedge c) \wedge \\
(y \leftrightarrow b \vee x) \wedge \\
(u \leftrightarrow a \vee b) \wedge \\
(v \leftrightarrow b \vee c) \wedge \\
(w \leftrightarrow u \wedge v) \wedge \\
(o \leftrightarrow y \oplus w)
\end{array}$$

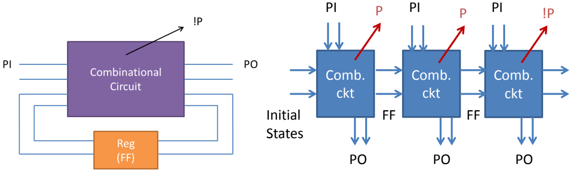
Optional: Bounded Model Checking

- We want to check property AG(p) for a given sequential circuit. See whether it has bugs!



Optional: Timeframe Expansion Model

- Iterative timeframe expansion model: sequential SAT becomes a combinational problem.

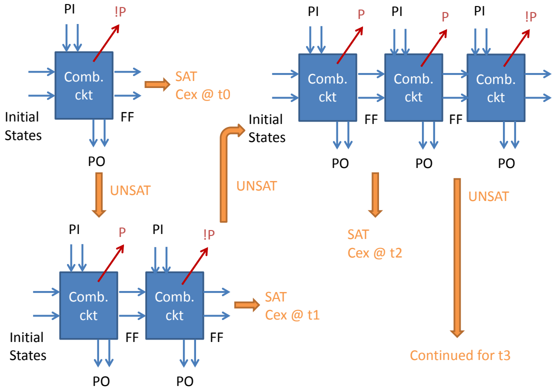


Optional: BMC Algorithm

- Let C be the set of constraints on the combinational circuit
- For an iterative model that unfolds the circuit for n times, let C_i correspond to the i-th iteration of the circuit constraint$(0 \leqq i \leqq k-1)$
- Let I_0 be the initial state value
- Let P be the property to prove
- Following is the BMC algorithm:
- BMC(P)
  - Let k=1
  - loop:
    - if (SAT($I_0 \wedge C_0 \wedge ... \wedge C_{k-1} \wedge !P_{k-1}$))
    -     return Find a counter-example at time (k-1)
    - k=k+1
    - go to loop

Optional: BMC Algorithm

- In other words ...



Solving Various Problems

- We now know how to use SAT.
- For many NP problems, SAT is a powerful tool. All you need is to develop proper SAT formulation, i.e. encoding your constraints into CNF formula
- Small Toy: online CNF generators