

## Homework Assignment #4

## Note

This assignment is due 2:10PM Wednesday, October 24, 2012. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building 2. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

## Problems

There are five problems in this assignment, each accounting for 20 points unless otherwise marked.

1. Identify the binding and bound occurrences of variable/function names in the following expressions.

(a) **let**  $x = 3$  **in**  
**let**  $sq\ x = x * x$  **in**  
 $sq\ x$

(b) **let rec**  $f\ x = g\ 1\ x$   
**and**  $g\ a\ x = x + a$  **in**  
**let**  $x = 3$  **in**  
 $f\ x$

2. Define a function  $f$  that satisfies the following requirement. (Define and use additional functions if needed.)

For  $x \geq 0$ ,  $f\ x$  is the largest integer  $n \geq 0$  such that  $n^2 \leq x$ .

3. An alternative to the Fibonacci function  $fib$  (in the previous homework assignment) is  $fast$  as defined below.

```
let rec  $g\ i\ j\ k\ n =$ 
  if  $k = n$  then  $j$ 
  else  $g\ j\ (i + j)\ (k + 1)\ n$ 
let  $fast\ n = g\ 0\ 1\ 0\ n$ 
```

Prove that  $fast\ n = fib\ n$  for all  $n \geq 0$ .

4. Explain why  $fast$  (in the preceding problem) is much more efficient than  $fib$  (in the previous homework assignment).
5. Extend the language of *Little Quilt* (discussed in class) so that functions behaving as suggested below can be defined.

$copy(1, \nabla) = \nabla$

$copy(3, \triangle \nabla) = \triangle \nabla \nabla \nabla$