

## Inference Rules of Hoare Logic

$$\frac{}{\{Q[E/x]\} x := E \{Q\}} \quad (\text{Assignment})$$

Note: to treat multiple assignments, view  $x$  as a list of distinct variables and  $E$  as a list of expressions.

$$\frac{}{\{Q[(b; i : E)/b]\} b[i] := E \{Q\}} \quad (\text{Assignment: array})$$

$$\frac{}{\{P\} \text{skip} \{P\}} \quad (\text{Skip})$$

$$\frac{\{P\} S_1 \{Q\} \quad \{Q\} S_2 \{R\}}{\{P\} S_1; S_2 \{R\}} \quad (\text{Sequence})$$

$$\frac{\{P \wedge B\} S_1 \{Q\} \quad \{P \wedge \neg B\} S_2 \{Q\}}{\{P\} \text{if } B \text{ then } S_1 \text{ else } S_2 \text{ fi} \{Q\}} \quad (\text{Conditional})$$

“if  $B$  then  $S$  fi” can be treated as “if  $B$  then  $S$  else skip fi” or directly with the following rule:

$$\frac{\{P \wedge B\} S \{Q\} \quad P \wedge \neg B \rightarrow Q}{\{P\} \text{if } B \text{ then } S \text{ fi} \{Q\}} \quad (\text{If-Then})$$

$$\frac{\{P \wedge B\} S \{P\}}{\{P\} \text{while } B \text{ do } S \text{ od} \{P \wedge \neg B\}} \quad (\text{while})$$

$$\frac{P \rightarrow P' \quad \{P'\} S \{Q'\} \quad Q' \rightarrow Q}{\{P\} S \{Q\}} \quad (\text{Consequence})$$

$$\frac{\text{“proc } p(\text{in } x; \text{in out } y; \text{out } z); \{P\} S \{Q\};\text{” is proved}}{\{P[a, b/x, y] \wedge I\} p(a, b, c) \{Q[b, c/y, z] \wedge I\}} \quad (\text{Procedure Call})$$

where  $b, c$  are (lists of) distinct variables and  $I$  does not refer to variables changed by procedure  $p$ .

$$\frac{\{P \wedge B\} S \{P\} \quad \{P \wedge B \wedge t = Z\} S \{t < Z\} \quad P \wedge B \rightarrow (t \geq 0)}{\{P\} \text{while } B \text{ do } S \text{ od} \{P \wedge \neg B\}} \quad (\text{while: simply total})$$

$$\frac{\{P \wedge B\} S \{P\} \quad \{P \wedge B \wedge \delta = D\} S \{\delta \prec D\} \quad P \wedge B \rightarrow (\delta \in W)}{\{P\} \text{while } B \text{ do } S \text{ od} \{P \wedge \neg B\}} \quad (\text{while: well-founded})$$

Auxiliary Rules:

$$\frac{P \rightarrow P' \quad \{P'\} S \{Q\}}{\{P\} S \{Q\}} \quad \text{(Strengthening Precondition)}$$

$$\frac{\{P\} S \{Q'\} \quad Q' \rightarrow Q}{\{P\} S \{Q\}} \quad \text{(Weakening Postcondition)}$$

$$\frac{\{P_1\} S \{Q_1\} \quad \{P_2\} S \{Q_2\}}{\{P_1 \wedge P_2\} S \{Q_1 \wedge Q_2\}} \quad \text{(Conjunction)}$$

$$\frac{\{P_1\} S \{Q_1\} \quad \{P_2\} S \{Q_2\}}{\{P_1 \vee P_2\} S \{Q_1 \vee Q_2\}} \quad \text{(Disjunction)}$$