

Alloy

(Based on [Jackson 2006])

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Outline

About Alloy

Logic

Language

Analysis

The Alloy Philosophy

- 🌐 The core of software development is the design of **abstractions**.
- 🌐 An abstraction is an idea reduced to its essential form.
- 🌐 You carefully design the abstractions and then develop their embodiments in code.
- 🌐 To find flaws early, the abstractions should be made precise and unambiguous using **formal specification**.
- 🌐 To be practically useful, the formal notation should be based on a small core of simple and robust concepts.
- 🌐 It is even more important to adopt a **fully automatic analysis** that provides immediate feedbacks.
- 🌐 The insist on full automation, according to the originator, was inspired by the success of model checking.



What Is Alloy?

- 🌐 The Alloy approach consists of a modeling language and an automatic analyzer.
- 🌐 The language, Alloy, is a structural modelling language based on **first-order logic**, for expressing complex structural constraints and behaviors.
- 🌐 The Alloy Analyzer is a constraint solver that provides fully automatic **simulation** and **checking**.
- 🌐 The approach is developed by the Software Design Group of Daniel Jackson at MIT.
- 🌐 Jackson boasts the approach to be “lightweight formal methods”.


- Like OCL, Alloy has a pure ASCII notation and does not require special typesetting tools.
- As a modeling language, Alloy is similar to OCL, but it has a more conventional syntax and a simpler semantics.
- Unlike OCL, Alloy is designed for fully automatic analysis.

Alloy = Logic + Language + Analysis




Logic

-  the core that provides the fundamental concepts
-  first-order logic + relational calculus

Language

-  syntax for structuring specifications in the logic

Analysis

-  bounded search by constraint solving
-  simulation: finding instances of states or executions that satisfy a given property
-  checking: finding a counterexample to a given property

Example

- 🌐 An address book for an email client
 - ☀️ associates email addresses with shorter names that are more convenient to use.
 - ☀️ alias: a nickname that can be used in place of the person's address
 - ☀️ group: an entire set of correspondents
- 🌐 Sample models under “book/chapter2” in the Alloy Analyzer

About Alloy

Logic

Language

Analysis

Three Logics in One

Predicate calculus style

Two kinds of expression: relation names, which are used as predicates, and tuples formed from quantified variables.

all n : Name, d, d' : Address |

$n \rightarrow d$ **in** address **and** $n \rightarrow d'$ **in** address **implies** $d = d'$

Navigation expression style (probably the most convenient)

Expressions denote sets, which are formed by “navigating” from quantified variables along relations.

all n : Name | **lone** n .address

Relational calculus style


Expressions denote relations, and there are no quantifiers at all.

no \sim address.address – **iden**

Atoms and Relations

- 🌐 **Atoms** are Alloy's primitive entities.
 - ☀️ They are indivisible, immutable, and uninterpreted.
- 🌐 A **relation** is a structure that relates atoms.
 - ☀️ It consists of a set of tuples, each tuple being a sequence of one or more atoms.
 - ☀️ All relations are first-order, i.e., relations cannot contain relations.
- 🌐 Every value in the Alloy logic is a relation.
 - ☀️ **Relations, sets, and scalars all are the same thing.**
 - ☀️ A scalar is represented by a singleton set.


Everything Is a Relation

 Sets are unary relations

Name = $\{(N0), (N1), (N2)\}$

Addr = $\{(A0), (A1), (A2)\}$

Book = $\{(B0), (B1)\}$

 Scalars are singleton sets (unary relation with only one tuple)

myName = $\{(N0)\}$

yourName = $\{(N2)\}$

myBook = $\{(B0)\}$

 Binary relation

name = $\{(B0, N0), (B1, N0), (B2, N2)\}$

 Ternary relation

addr = $\{(B0, N0, A0), (B0, N1, A1),$
 $(B1, N1, A2), (B1, N2, A2)\}$

Constants

- none** empty set
univ universal set
iden identity

Example

Name = {(N0), (N1), (N2)}

Addr = {(A0), (A1)}

none = {}

univ = {(N0), (N1), (N2), (A0), (A1)}

iden = {(N0, N0), (N1, N1), (N2, N2), (A0, A0), (A1, A1)}

Set Operators

Example

Name = {(N0), (N1), (N2)}

Alias = {(N1), (N2)}

Group = {(N0)}

RecentlyUsed = {(N0), (N2)}

- + union
- & intersection
- difference
- in** subset
- = equality

Alias + Group = {(N0), (N1), (N2)}

Alias & RecentlyUsed = {(N2)}

Name - RecentlyUsed = {(N1)}

RecentlyUsed **in** Alias = false

RecentlyUsed **in** Name = true

Name = Group + Alias = true

Product Operator

-> arrow (product)

Example

Name = {(N0), (N1)}

Addr = {(A0), (A1)}

Book = {(B0)}

Name->Addr = {(N0, A0), (N0, A1), (N1, A0), (N1, A1)}

Book->Name->Addr =
{(B0, N0, A0), (B0, N0, A1), (B0, N1, A0), (B0, N1, A1)}

Relational Join

$$p \cdot q \equiv \begin{array}{|c|} \hline (a, b) \\ \hline (a, c) \\ \hline (b, d) \\ \hline \end{array} \cdot \begin{array}{|c|} \hline (a, d, c) \\ \hline (b, c, c) \\ \hline (c, c, c) \\ \hline (b, a, d) \\ \hline \end{array} = \begin{array}{|c|} \hline (a, c, c) \\ \hline (a, a, d) \\ \hline \end{array}$$

$$x \cdot f \equiv \begin{array}{|c|} \hline (c) \\ \hline \end{array} \cdot \begin{array}{|c|} \hline (a, b) \\ \hline (b, d) \\ \hline (c, a) \\ \hline (d, a) \\ \hline \end{array} = \begin{array}{|c|} \hline (a) \\ \hline \end{array}$$

Join Operators

- dot (join)
- [] box (join)

$$e1[e2] = e2.e1$$

$$a.b.c[d] = d.(a.b.c)$$

Example

Book = {(B0)}
 Name = {(N0), (N1), (N2)} myName = {(N1)}
 Addr = {(A0), (A1), (A2)} myAddr = {(A0)}
 Host = {(H0), (H1)}
 address = {(B0, N0, A0), (B0, N1, A0), (B0, N2, A2)}
 host = {(A0, H0), (A1, H1), (A2, H1)}

 Book.address = {(N0, A0), (N1, A0), (N2, A2)}
 Book.address[myName] = {(A0)}
 Book.address.myName = {}
 host[myAddr] = {(H0)}
 address.host = {(B0, N0, H0), (B0, N1, H0), (B0, N2, H1)}

Unary Operators

- ~ transpose
- ^ transitive closure
- * reflexive transitive closure
(apply only to binary relations)

$$\hat{r} = r + r.r + r.r.r + \dots$$

$$*r = \mathbf{idem} + \hat{r}$$

Example

Node = {(N0), (N1), (N2), (N3)}

first = {(N0)} next = {(N0, N1), (N1, N2), (N2, N3)}

~next = {(N1, N0), (N2, N1), (N3, N2)}

^next = {(N0, N1), (N0, N2), (N0, N3),
(N1, N2), (N1, N3), (N2, N3)}

*next = {(N0, N0), (N0, N1), (N0, N2), (N0, N3), (N1, N1),
(N1, N2), (N1, N3), (N2, N2), (N2, N3), (N3, N3)}

first.^next = {(N1), (N2), (N3)}

first.*next = Node

Restriction and Override

- <: domain restriction
- :> range restriction
- ++ override

$$p ++ q =$$

$$p - (\text{domain}[q] <: p) + q$$

Example

Name = {(N0), (N1), (N2)}

Alias = {(N0), (N1)} Addr = {(A0)}

address = {(N0, N1), (N1, N2), (N2, A0)}

address :> Addr = {(N2, A0)}

Alias <: address = {(N0, N1), (N1, N2)}

address :> Name = {(N0, N1), (N1, N2)}

address :> Alias = {(N0, N1)}

workAddress = {(N0, N1), (N1, A0)}

address ++ workAddress = {(N0, N1), (N1, A0), (N2, A0)}

$m' = m ++ (k \rightarrow v)$ *update map m with key-value pair (k, v)*

Boolean Operators

not	!	negation
and	&&	conjunction
or		disjunction
implies	=>	implication
else		alternative
iff	<=>	bi-implication

Example

Four equivalent constraints:

$F \Rightarrow G$ **else** H

F **implies** G **else** H

$(F \ \&\& \ G) \ || \ ((\text{not } F) \ \&\& \ H)$

$(F \ \text{and} \ G) \ \text{or} \ ((\text{not } F) \ \text{and} \ H)$

Quantification

- all** $x: e \mid F$ F holds for *every* x in e
- some** $x: e \mid F$ F holds for *at least one* x in e
- no** $x: e \mid F$ F holds for *no* x in e
- lone** $x: e \mid F$ F holds for *at most one* x in e
- one** $x: e \mid F$ F holds for *exactly one* x in e

Example

some n : Name, a : Address $\mid a$ **in** n .address

some name maps to some address - address book not empty

no n : Name $\mid n$ **in** n . $\hat{}$ address

no name can be reached by lookups from itself - address book acyclic

all n : Name \mid **lone** a : Address $\mid a$ **in** n .address

every name maps to at most one address - address book is functional

all n : Name \mid **no disj** a, a' : Address $\mid (a + a')$ **in** n .address

no name maps to two or more distinct addresses - same as above

Quantified Expressions

some e e has *at least one* tuple

no e e has *no* tuples

lone e e has *at most one* tuple

one e e has *exactly one* tuple

Example

some Name

set of names is not empty

some address

address book is not empty - it has a tuple

no (address.Addr - Name)

nothing is mapped to addresses except names

all n: Name | **lone** n.address

every name maps to at most one address

Let Expressions and Constraints

let $x = e \mid A$

f implies e1 else e2

A can be a constraint or an expression.

if f then e1 else e2

Example

Four equivalent constraints:

all $n: \text{Name} \mid (\text{some } n.\text{workAddress}$
implies $n.\text{address} = n.\text{workAddress}$ **else** $n.\text{address} = n.\text{homeAddress})$

all $n: \text{Name} \mid \text{let } w = n.\text{workAddress}, a = n.\text{address} \mid$
(some w **implies** $a = w$ **else** $a = n.\text{homeAddress})$

all $n: \text{Name} \mid \text{let } w = n.\text{workAddress} \mid$
 $n.\text{address} = (\text{some } w$ **implies** w **else** $n.\text{homeAddress})$

all $n: \text{Name} \mid n.\text{address} =$
(let $w = n.\text{workAddress} \mid (\text{some } w$ **implies** w **else** $n.\text{homeAddress}))$

Comprehensions

$$\{x_1: e_1, x_2: e_2, \dots, x_n: e_n \mid F\}$$

Example

$$\{n: \text{Name} \mid \mathbf{no} \ n. \hat{\text{address}} \ \& \ \text{Addr}\}$$


set of names that don't resolve to any actual addresses

$$\{n: \text{Name}, a: \text{Address} \mid n \rightarrow a \ \mathbf{in} \ \hat{\text{address}}\}$$

binary relation mapping names to reachable addresses

Declarations

relation-name : expression

 almost the same as the meaning of a subset constraint $x \mathbf{in} e$

Example

address: Name \rightarrow Addr

a single address book mapping names to addresses

addr: Book \rightarrow Name \rightarrow Addr

a collection of address books mapping books to names to addresses

address: Name \rightarrow (Name + Addr)

a multilevel address book mapping names to names and addresses

Set Multiplicities

set any number
one exactly one
lone zero or one
some one or more

x: *m* e

x: e \Leftrightarrow x: **one** e

Example

RecentlyUsed: **set** Name

RecentlyUsed is a subset of the set Name

senderAddress: Addr

senderAddress is a singleton subset of Addr

senderName: **lone** Name

senderName is either empty or a singleton subset of Name


receiverAddresses: **some** Addr


receiverAddresses is a nonempty subset of Addr


Relation Multiplicities

$r: A \ m \rightarrow \ n \ B$

 $r: A \ m \rightarrow \ n \ B \Leftrightarrow ((\mathbf{all} \ a: A \ | \ n \ a.r) \ \mathbf{and} \ (\mathbf{all} \ b: B \ | \ m \ r.b))$

 $r: A \rightarrow B \Leftrightarrow r: A \ \mathbf{set} \rightarrow \ \mathbf{set} \ B$

 $r: A \rightarrow (B \ m \rightarrow \ n \ C) \Leftrightarrow \mathbf{all} \ a: A \ | \ a.r: B \ m \rightarrow \ n \ C$

 $r: (A \ m \rightarrow \ n \ B) \rightarrow C \Leftrightarrow \mathbf{all} \ c: C \ | \ r.c: A \ m \rightarrow \ n \ B$

Example

workAddress: Name \rightarrow **lone** Addr

each name refers to at most one work address

members: Group **lone** \rightarrow **some** Addr

address belongs to at most one group name and group contains at least one address

Cardinality Constraints

#r	number of tuples in r	=	equals
0, 1, ...	integer literal	<	less than
+	plus	>	greater than
-	minus	=<	less than or equal to
		>=	greater than or equal to

sum x: e | ie

sum of integer expression *ie* for all singletons *x* drawn from *e*

Example

all b: Bag | #b.marbles =< 3

all bags have 3 or less marbles

#Marble = **sum** b: Bag | #b.marbles

the sum of the marbles across all bags equals the total number of marbles

About Alloy

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“I’m My Own Grandpa” in Alloy

```
module language/grandpa1 /* module header */  
abstract sig Person { /* signature declarations */  
  father: lone Man,  
  mother: lone Woman  
}  
sig Man extends Person {  
  wife: lone Woman  
}  
sig Woman extends Person {  
  husband: lone Man  
}  
fact { /* constraint paragraphs */  
  no p: Person | p in p.^(mother + father)  
  wife = ~husband  
}
```

"I'm My Own Grandpa" in Alloy (Cont'd)

```
assert noSelfFather { /* assertions */  
    no m: Man | m = m.father  
}  
check noSelfFather /* commands */  
  
fun grandpas[p: Person] : set Person { /* constraint paragraphs */  
    p.(mother + father).father  
}  
pred ownGrandpa[p: Person] { /* constraint paragraphs */  
    p in grandpas[p]  
}  
run ownGrandpa for 4 Person /* commands */
```

“I’m My Own Grandpa” in Alloy (Cont’d)

module language/grandpa3

...

fact {

no p: Person | p **in** p.^(mother + father) /* biology */

wife = ~husband /* terminology */

no (wife + husband) & ^ (mother + father) /* social convention */

}

...

fun grandpas[p: Person] : set Person {

let parent = mother + father + father.wife + mother.husband |

p.parent.parent & Man

}

pred ownGrandpa[p: Person] {

p **in** grandpas[p]

}

run ownGrandpa **for** 4 Person

sig A {}

set of atoms A

sig A {}

sig B {}

disjoint sets A and B (no A & B)

sig A, B {}

same as above

Signatures (Cont'd)

sig B extends A {}

set B is a subset of A (B in A)

sig B extends A {}

sig C extends A {}

B and C are disjoint subsets of A (B in A && C in A && no B & C)

sig B, C extends A {}

same as above

abstract sig A {}

sig B extends A {}

sig C extends A {}

A is partitioned by disjoint subsets B and C (no B & C && A = (B + C))

Signatures (Cont'd)

sig B in A { }

B is a subset of A - not necessarily disjoint from any other set

sig C in A + B { }

C is a subset of the union of A and B

one sig A { }

lone sig B { }

some sig C { }

A is a singleton set

B is a singleton or empty

C is a non-empty set

Field Declarations

sig A {f: e}

f is a binary relation with domain A and range given by expression e

f is constrained to be a function: (f: A \rightarrow **one** e) or (**all** a: A | a.f: **one** e)

sig A { f1: **one** e1, f2: **lone** e2, f3: **some** e3, f4: **set** e4 }

(all a: A | a.fn : m e)

sig A {f, g: e}

two fields with same constraints

sig A {f: e1 m \rightarrow n e2}

(f: A \rightarrow (e1 m \rightarrow n e2)) or (**all** a: A | a.f : e1 m \rightarrow n e2)

sig Book {

names: set Name,





addrs: names \rightarrow Addr

}

dependent fields (**all** b: Book | b.addrs: b.names \rightarrow Addr)

Fields in the “Self-Grandpas” Example

```
abstract sig Person {  
  father: lone Man,  
  mother: lone Woman  
}  
sig Man extends Person {  
  wife: lone Woman  
}  
sig Woman extends Person {  
  husband: lone Man  
}
```


-  Fathers are men and everyone has at most one.
-  Mothers are women and everyone has at most one.
-  Wives are women and every man has at most one.
-  Husbands are men and every woman has at most one.

Facts

fact { F }

fact f { F }

sig S { ... } { F }

 Facts introduce constraints that are assumed to always hold.

Example

sig Host { }

sig Link {from, to: Host}

fact {**all** x: Link | x.from != x.to}

no links from a host to itself

fact noSelfLinks {**all** x: Link | x.from != x.to}




same as above

sig Link {from, to: Host} {from != to}

same as above, with implicit 'this.'

Facts in “Self-Grandpas”

```
fact {  
  no p: Person |  
    p in p.^(mother + father)  
  wife = ~husband  
}
```

-  No person is his or her own ancestor.
-  A man's wife has that man as a husband.
-  A woman's husband has that woman as a wife.

Functions

fun $f[x_1: e_1, \dots, x_n: e_n] : e \{ E \}$

- 🌐 Functions are named expressions with declaration parameters and a declaration expression as a result invoked by providing an expression for each parameter.

Example

```
sig Name, Addr { }
```


```
sig Book { addr: Name -> Addr }
```

```
fun lookup[b: Book, n: Name] : set Addr {  
  b.addr[n]  
}
```

```
fact everyNameMapped {  
  all b: Book, n: Name | some lookup[b, n]  
}
```

Predicates

pred $p[x_1: e_1, \dots, x_n: e_n] \{ F \}$

 Predicates are named formulae with declaration parameters.

Example

```
sig Name, Addr { }
```


```
sig Book { addr: Name -> Addr }
```

```
pred contains[b: Book, n: Name, d: Addr] {  
  n->d in b.addr  
}
```

```
fact everyNameMapped {  
  all b: Book, n: Name |  
    some d: Addr | contains[b, n, a]  
}
```


Functions and Predicates in “Self-Grandpas”

```
fun grandpas[p: Person] : set Person {  
    p.(mother + father).father  
}  
  
pred ownGrandpa[p: Person] {  
    p in grandpas[p]  
}
```

 A person's grandpas are the fathers of one's own mother and father.


“Receiver” Syntax

fun f[x: X, y: Y, ...] : Z {...x...}

fun X.f[y:Y, ...] : Z {...this...}

pred p[x: X, y: Y, ...] {...x...}

pred X.p[y:Y, ...] {...this...}

 Whether or not the predicate or function is declared in this way, it can be used in the form

x.p[y, ...]

where x is taken as the first argument, y as the second, and so on.


Example

```
fun Person.grandpas : set Person {  
  this.(mother + father).father  
}
```

```
pred Person.ownGrandpa {  
  this in this.grandpas  
}
```

Assertions

assert a { F }

 An assertion is a constraint intended to follow from facts of the model.



Example

```
sig Node {children: set Node}
one sig Root extends Node {}
fact { Node in Root.*children }
assert someParent { // invalid assertion
  all n: Node | some children.n
}
assert someParent { // valid assertion
  all n: Node - Root | some children.n
}
```

Check Commands

assert a { F }

check a *scope*

-  instructs the analyzer to search for a counterexample to assertion within the scope
-  if the model has facts M , finds a solution to $M \&\&!F$

Example

check a

top-level sigs bound by 3

check a **for** *default*

top-level sigs bound by default

check a **for** *default* **but** *list*

default overridden by bounds in list

check a **for** *list*

sigs bound in list, invalid if any unbound


Check Commands (Cont'd)

Example

```
abstract sig Person {}  
sig Man extends Person {}  
sig Woman extends Person {}  
sig Grandpa extends Man {}  
  
check a  
check a for 4  
check a for 4 but 3 Man, 5 Woman  
check a for 4 Person  
check a for 3 Man, 4 Woman  
check a for 3 Man, 4 Woman, 2 Grandpa  
  
// invalid, because top-level bounds unclear  
check a for 3 Man  
check a for 5 Woman, 2 Grandpa
```

Assertion Checks in “Self-Grandpas”



```
fact {  
  no p: Person | p in p.^(mother + father)  
  wife = ~husband  
}  
  
assert noSelfFather {  
  no m: Man | m = m.father  
}  
  
check noSelfFather
```

-  The check command instructs the analyzer to search for a counterexample to noSelfFather within a scope of at most 3 Persons.

Run Commands



pred $p[x: X, y: Y, \dots] \{ F \}$

run p *scope*

-  instructs the analyzer to search for an instance of the predicate within scope
-  if the model has facts M , finds a solution to $M \ \&\& \ (some \ x : X, y : Y, \dots \mid F)$


fun $f[x: X, y: Y, \dots] : R \{ E \}$

run f *scope*

-  instructs the analyzer to search for an instance of the function within scope
-  if the model has facts M , finds a solution to $M \ \&\& \ (some \ x : X, y : Y, \dots, result : R \mid result = E)$

Predicate Simulation in “Self-Grandpas”

```
fun grandpas[p: Person] : set Person {  
  p.(mother + father).father  
}  
  
pred ownGrandpa[p: Person] {  
  p in grandpas[p]  
}  
  
run ownGrandpa for 4 Person
```

-  The run command instructs the analyzer to search for a configuration with at most 4 people in which a man is his own grandfather.

Types and Type Checking

- Alloy's type system has two functions.
 - It allows the analyzer to catch errors before any serious analysis is performed.
 - It is used to resolve overloading.
- A *basic type* is introduced for each top-level signature and for each extension signature.
 - A signature that is declared independently of any other is a *top-level* signature.
- When signature $A1$ extends signature A , the type associated with $A1$ is a *subtype* of the type associated with A .
- A subset signature acquired its parent's type.
 - If declared as a subset of a union of signatures, its type is the union of the types of its parents.
- Two basic types are said to *overlap* if one is a subtype of the other.

Types and Type Checking (Cont'd)

- Every expression has a *relational type*, consisting of a union of products:

$$A_1 \rightarrow B_1 \rightarrow \dots + A_2 \rightarrow B_2 \rightarrow \dots + \dots$$

where each of the A_i , B_i , and so on, is a basic type.

- A binary relation's type, for example, will look like this:

$$A_1 \rightarrow B_1 + A_2 \rightarrow B_2 + \dots$$

and a set's type like this:

$$A_1 + A_2 + \dots$$

- The type of an expression is itself just an Alloy expression.
- Types are inferred automatically so that the value of the type always contains the values of the expressions. It's an *overapproximation*.
 - If two types have an empty intersection, the expressions they were obtained from must also have an empty intersection.

Types and Type Checking (Cont'd)

- 🌐 There are two kinds of type error.
 - ☀ It is illegal to form expressions that would give relations of mixed arity.
 - ☀ An expression is illegal if it can be shown, from the declarations alone, to be redundant, or to contain a redundant subexpression.
- 🌐 The subtype hierarchy is used primarily to determine whether types are disjoint.
- 🌐 The typing of an expression of the form $s.r$ where s is a set and r is a relation only requires s and the domain of r to overlap.
 - ☀ The case that two types are disjoint is rejected, because it always results in the empty set.
- 🌐 Type checking is sound.
 - ☀ When checking an intersection expression, for example, if the resulting type is empty, the relation represented by the expression must be empty.

Types and Type Checking (Cont'd)

- 🌐 A signature defines a local namespace for its declarations, so you can use the same field name in different signatures.
- 🌐 When a field name refers to possibly multiple fields, the types of the candidate fields are used to determine which field is meant.
- 🌐 If more than one field is possible, an error is reported.

Example

```
sig Object, Block {}
```

```
sig Directory extends Object {contents: set Object}
```

```
sig File extends Object {contents: set Block}
```

```
all f: File | some f.contents
```

```
// The occurrence of the field name contents is trivially resolved.
```

```
all o: Object | some o.contents
```

```
// The occurrence of contents here is not resolved, and the constraint is rejected.
```

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The Alloy Analyzer

- 🌐 The Alloy Analyzer is a 'model finder'.
- 🌐 Given a logical formula, it attempts to find a model that makes the formula true.
 - ☀️ A model is a binding of the variables to values.
- 🌐 For **simulation**, the formula will be some part of the system description.
 - ☀️ If it is a state invariant INV, models of INV will be states that satisfy the invariant.
 - ☀️ If it is an operation OP, with variables representing the before and after states, models of OP will be legal state transitions.
- 🌐 For **checking**, the formula is a negation, usually of an implication.
 - ☀️ To check that the system described by the property SYS has a property PROP, you would assert (SYS implies PROP).
 - ☀️ The Alloy Analyzer negates the assertion, and looks for a model of (SYS and not PROP), which, if found, will be a counterexample to the claim.

The Small Scope Hypothesis

- Simulation is for determining consistency (i.e., satisfiability) and checking is for determining validity and these problems are undecidable for Alloy specifications.
- The Alloy Analyzer restricts the simulation and checking operations to a finite scope.
- The validity and consistency problems within a finite scope are decidable problems.
- Most bugs have a small counterexample.*
- If an assertion is invalid, it probably has a small counterexample.

How Does It Work

- 🌐 The Alloy Analyzer is essentially a compiler.
- 🌐 It translates the problem to be analyzed into a (usually huge) boolean formula.
- 🌐 Think about a particular value of a binary relation r from a set A to a set B :
 - ☀️ The value can be represented as an adjacency matrix of 0's and 1's, with a 1 in row i and column j when the i th element of A is mapped to the j th element of B .
 - ☀️ So the space of all possible values of r can be represented by a matrix of *boolean variables*.
 - ☀️ The dimensions of these matrices are determined by the scope; if the scope bounds A by 3 and B by 4, r will be a 3×4 matrix containing 12 boolean variables, and having 2^{12} possible values.

How Does It Work (Cont'd)

- Now, for each relational expression, a matrix is created whose elements are boolean expressions.
 - For example, the expression corresponding to $p + q$ for binary relations p and q would have the expression $p_{i,j} \vee q_{i,j}$ in row i and column j .
- For each relational formula, a boolean formula is created.
 - For example, the formula corresponding to p **in** q would be the conjunction of $p_{i,j} \Rightarrow q_{i,j}$ over all values of i and j .
- The resulting formula is handed to a SAT solver, and the solution is translated back by the Alloy Analyzer into the language of the model.
- All problems are solved within a user-specified scope that bounds the size of the domains, and thus makes the problem finite (and reducible to a boolean formula).
- Alloy analyzer implements an efficient translation in the sense that the problem instance presented to the SAT solver is as small as possible.

Differences from Model Checkers

- 🌐 The Alloy Analyzer is designed for analyzing state machines with operations over complex states.
- 🌐 Model checkers are designed for analyzing state machines that are composed of several state machines running in parallel, each with relatively simple states.
- 🌐 Alloy allows structural constraints on the state to be described very directly (with sets and relations), whereas most model checking languages provide only relatively low-level data types (such as arrays and records).
- 🌐 Model checkers do a temporal analysis that compares a state machine to another machine or a temporal logic formula.