

Automata-Based Model Checking

(Based on [Clarke et al. 1999] and [Holzmann 2003])

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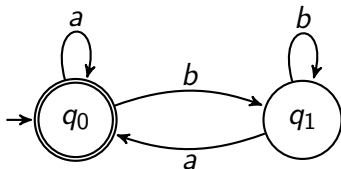
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Büchi Automata

- The simplest computation model for **finite** behaviors is the **finite state automaton**, which accepts finite words.
- The simplest computation model for **infinite** behaviors is the **ω -automaton**, which accepts infinite words.
- Both have the same syntactic structure.
- Model checking traditionally deals with **non-terminating** concurrent systems.
- Infinite words conveniently represent the infinite behaviors exhibited by a non-terminating system.
- **Büchi automata** are the simplest kind of ω -automata.
- They were first proposed and studied by J.R. Büchi in the early 1960's.

An Example Büchi Automaton



- A Büchi automaton accepts an infinite word if the word drives the automaton through some accepting state infinitely many times.
- The above Büchi automaton accepts infinite words over $\{a, b\}$ that have infinitely many a 's.
- Using an ω -regular expression, its language is expressed as $(b^*a)^\omega$.

Büchi Automata (cont.)

- Formally, a **Büchi automaton (BA)**, like a finite-state automaton (FA), is given by a 5-tuple $(\Sigma, Q, \Delta, q_0, F)$:
 - Σ is a finite set of symbols (the *alphabet*),
 - Q is a finite set of *states*,
 - $\Delta \subseteq Q \times \Sigma \times Q$ is the *transition relation*,
 - $q_0 \in Q$ is the *start* (or *initial*) state (sometimes we allow multiple start states, indicated by Q_0 or Q^0), and
 - $F \subseteq Q$ is the set of *accepting* (final in FA) states.
- Let $B = (\Sigma, Q, \Delta, q_0, F)$ be a BA and $w = w_1 w_2 \dots w_i w_{i+1} \dots$ be an infinite string (or word) over Σ .
- A *run* of B over w is a sequence of states $r_0, r_1, r_2, \dots, r_i, r_{i+1}, \dots$ such that
 - $r_0 = q_0$ and
 - $(r_i, w_{i+1}, r_{i+1}) \in \Delta$ for $i \geq 0$.

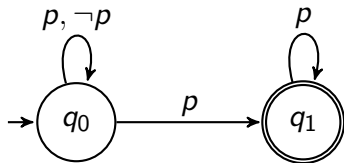
Büchi Automata (cont.)

- Let $\text{inf}(\rho)$ denote the set of states occurring infinitely many times in a run ρ .
- A run ρ is *accepting* if it satisfies the following condition:

$$\text{inf}(\rho) \cap F \neq \emptyset.$$

- An infinite word $w \in \Sigma^\omega$ is *accepted* by a BA B if there exists an accepting run of B over w .
- The *language* recognized by B (or the language of B), denoted $L(B)$, is the set of all words accepted by B .

Another Example



- This Büchi automaton has $\{p, \neg p\}$ as its alphabet.
- It accepts infinite words/sequences over $\{p, \neg p\}$ that eventually remain p forever.
- Its language corresponds to the set of sequences that satisfy the temporal formula $\diamond \square p$.

Closure Properties

- 🌐 A class of languages is **closed** under intersection if the intersection of any two languages in the class remains in the class.
- 🌐 Analogously, for closure under complementation.

Theorem

*The class of languages recognizable by Büchi automata is closed under **intersection** and **complementation** (and hence all boolean operations).*

- 🌐 Note: the theorem would not hold if we were restricted to *deterministic* Büchi automata, unlike in the classic case.

Generalized Büchi Automata

- 🌐 A **generalized Büchi automaton (GBA)** has an acceptance component of the form $F = \{F_1, F_2, \dots, F_n\} \subseteq 2^Q$.
- 🌐 A run ρ of a GBA is accepting if for each $F_i \in F$, $\text{inf}(\rho) \cap F_i \neq \emptyset$.
- 🌐 GBA's naturally arise in the modeling of finite-state concurrent systems with fairness constraints.
- 🌐 They are also a convenient intermediate representation in the translation from a linear temporal formula to an equivalent BA.
- 🌐 There is a simple translation from a GBA to a Büchi automaton, as shown next.

GBA to BA

- 🌐 Let $B = (\Sigma, Q, \Delta, q_0, F)$, where $F = \{F_1, \dots, F_n\}$, be a GBA.
- 🌐 Construct $B' = (\Sigma, Q \times \{0, \dots, n\}, \Delta', \langle q_0, 0 \rangle, Q \times \{n\})$.
- 🌐 The transition relation Δ' is constructed such that $(\langle q, x \rangle, a, \langle q', y \rangle) \in \Delta'$ when $(q, a, q') \in \Delta$ and x and y are defined according to the following rules:
 - ☀ If $q' \in F_i$ and $x = i - 1$, then $y = i$.
 - ☀ If $x = n$, then $y = 0$.
 - ☀ Otherwise, $y = x$.
- 🌐 Claim: $L(B') = L(B)$.

Theorem

For every GBA B , there is an equivalent BA B' such that $L(B') = L(B)$.

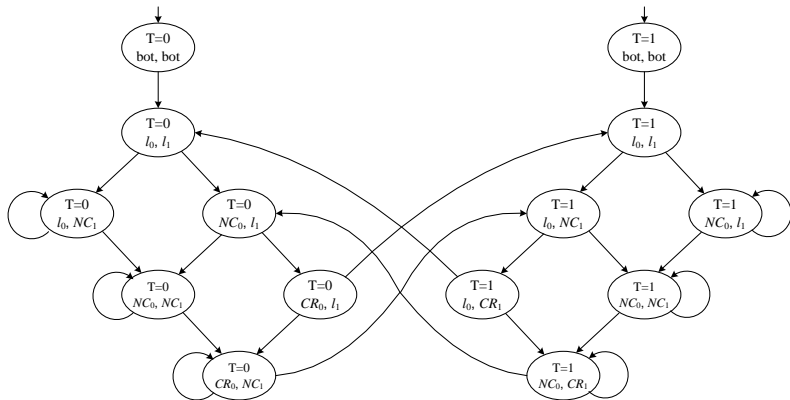
The Model Checking Problem

- Let AP be a set of atomic propositions.
- A **Kripke structure** M over AP is a 4-tuple $M = (S, R, S_0, L)$:
 - S is a finite set of states.
 - $R \subseteq S \times S$ is a transition relation that must be total, that is, for every state $s \in S$ there is a state $s' \in S$ such that $R(s, s')$.
 - $S_0 \subseteq S$ is the set of initial states.
 - $L : S \rightarrow 2^{AP}$ is a function that labels each state with the set of atomic propositions true in that state.
- A **computation** or **path** of M from a state s is an infinite sequence of states $\sigma = s_0, s_1, s_2, \dots$ such that $s_0 \in S_0$ and $(s_i, s_{i+1}) \in R$, for all $i \geq 0$.
- The **Model Checking** problem is to determine if the computations from the initial states of a Kripke structure M satisfy a property φ expressed as a temporal formula, i.e., if $M \models \varphi$.

A Mutual Exclusion Program

$$P_{MX} = m : \mathbf{cobegin} P_0 \parallel P_1 \mathbf{coend} m'$$
 $P_0 =$ $l_0 : \mathbf{while} \textit{True} \mathbf{do}$
 $NC_0 : \mathbf{wait} T = 0;$
 $CR_0 : T := 1;$
 $\mathbf{od};$ l'_0 $P_1 =$ $l_1 : \mathbf{while} \textit{True} \mathbf{do}$
 $NC_1 : \mathbf{wait} T = 1;$
 $CR_1 : T := 0;$
 $\mathbf{od};$ l'_1

Kripke Structure of the Program P_{MX}



The value of the outer program counter is not shown. Initially, the program counters of both processes have the value bot (\perp), indicating that they are not started yet.

Model Checking Using Automata

- Finite automata can be used to model concurrent and reactive systems as well.
- One of the main advantages of using automata for model checking is that both the **modeled system** and the **specification** are represented **in the same way**.
- A Kripke structure directly corresponds to a Büchi automaton, where all the states are accepting.
- A Kripke structure (S, R, S_0, L) can be transformed into an automaton $A = (\Sigma, S \cup \{\iota\}, \Delta, \iota, S \cup \{\iota\})$ with $\Sigma = 2^{AP}$ where
 - $(s, \alpha, s') \in \Delta$ for $s, s' \in S$ iff $(s, s') \in R$ and $\alpha = L(s')$ and
 - $(\iota, \alpha, s) \in \Delta$ iff $s \in S_0$ and $\alpha = L(s)$.

Model Checking Using Automata (cont.)

- The given system is modeled as a Büchi automaton A .
- Suppose the desired property is originally given by a linear temporal formula f .
- Let B_f (resp. $B_{\neg f}$) denote a Büchi automaton equivalent to f (resp. $\neg f$); we will later study how a temporal formula can be translated into an automaton.
- The model checking problem $A \models f$ is equivalent to asking whether

$$L(A) \subseteq L(B_f) \text{ or } L(A) \cap L(B_{\neg f}) = \emptyset.$$

- The well-used model checker SPIN, for example, adopts this automata-theoretic approach.
- So, we are left with two basic problems:
 - ☀ Compute the intersection of two Büchi automata.
 - ☀ Test the emptiness of the resulting automaton.

Intersection of Büchi Automata

- 🌐 Let $B_1 = (\Sigma, Q_1, \Delta_1, Q_1^0, F_1)$ and $B_2 = (\Sigma, Q_2, \Delta_2, Q_2^0, F_2)$.
- 🌐 We can build an automaton for $L(B_1) \cap L(B_2)$ as follows.
- 🌐 $B_1 \otimes B_2 =$
 $(\Sigma, Q_1 \times Q_2 \times \{0, 1, 2\}, \Delta, Q_1^0 \times Q_2^0 \times \{0\}, Q_1 \times Q_2 \times \{2\})$.
- 🌐 We have $(\langle r, q, x \rangle, a, \langle r', q', y \rangle) \in \Delta$ iff the following conditions hold:
 - ☀️ $(r, a, r') \in \Delta_1$ and $(q, a, q') \in \Delta_2$.
 - ☀️ The third component is affected by the accepting conditions of B_1 and B_2 .
 - 👤 If $x = 0$ and $r' \in F_1$, then $y = 1$.
 - 👤 If $x = 1$ and $q' \in F_2$, then $y = 2$.
 - 👤 If $x = 2$, then $y = 0$.
 - 👤 Otherwise, $y = x$.
- 🌐 The third component is responsible for guaranteeing that accepting states from both B_1 and B_2 appear infinitely often.

Intersection of Büchi Automata (cont.)

- 🌐 A simpler intersection may be obtained when all of the states of one of the automata are accepting.
- 🌐 Assuming all states of B_1 are accepting and that the acceptance set of B_2 is F_2 , their intersection can be defined as follows:

$$B_1 \otimes B_2 = (\Sigma, Q_1 \times Q_2, \Delta', Q_1^0 \times Q_2^0, Q_1 \times F_2)$$

where $(\langle r, q \rangle, a, \langle r', q' \rangle) \in \Delta'$ iff $(r, a, r') \in \Delta_1$ and $(q, a, q') \in \Delta_2$.

Checking Emptiness

- Let ρ be an accepting run (if one exists) of a Büchi automaton $B = (\Sigma, Q, \Delta, Q^0, F)$.
- In the context of model checking, the accepting run ρ , if found, represents a *counterexample* showing that the system does not satisfy the property.
- By definition, ρ contains infinitely many accepting states from F .
- Since Q is finite, there is some suffix ρ' of ρ such that every state on it appears infinitely many times.
- Each state on ρ' is reachable from any other state on ρ' .
- Hence, the states in ρ' are included in a (nontrivial) **strongly connected component**.
- This component is reachable from an initial state and contains an accepting state.

Checking Emptiness (cont.)

- Conversely, any strongly connected component that is reachable from an initial state and contains an accepting state generates an accepting run of the automaton.
- Thus, checking nonemptiness of $L(B)$ is equivalent to finding a strongly connected component that is reachable from an initial state and contains an accepting state.
- That is, the language $L(B)$ is nonempty iff there is a reachable accepting state with a cycle back to itself.

Double DFS Algorithm

```
procedure emptiness  
  for all  $q_0 \in Q^0$  do  
    dfs1( $q_0$ );  
  terminate(True);  
end procedure
```

```
procedure dfs1( $q$ )  
  local  $q'$ ;  
  hash( $q$ );  
  for all successors  $q'$  of  $q$  do  
    if  $q'$  not in the hash table then dfs1( $q'$ );  
  if accept( $q$ ) then dfs2( $q$ );  
end procedure
```

Double DFS Algorithm (cont.)

```
procedure dfs2(q)  
  local q';  
  flag(q);  
  for all successors q' of q do  
    if q' on dfs1 stack then terminate(False);  
    else if q' not flagged then dfs2(q');  
    end if;  
end procedure
```

Basic Practical Details

- 🌐 We now have the essential automata-based theory for model checking, but we still need to pay attention to a few more basic practical details.
- 🌐 Many systems are more naturally represented as the parallel composition of several concurrently executing processes, rather than as a monolithic chunk of code.
- 🌐 There are also concerns with the size of the system and the gap between the computation model and a concurrent system running on real hardware.
- 🌐 Specifically, we will look into
 - ☀ asynchronous products of automata,
 - ☀ on-the-fly state exploration, and
 - ☀ fairness (in the computation model).

Processes as Automata

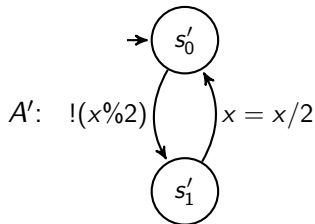
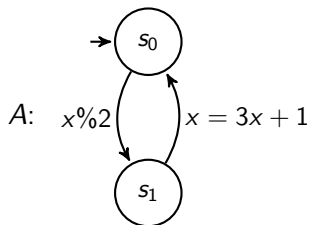
```

#define N 4
int x = N;

active proctype A0()
{
  do
    :: x%2 -> x = 3*x + 1
  od
}

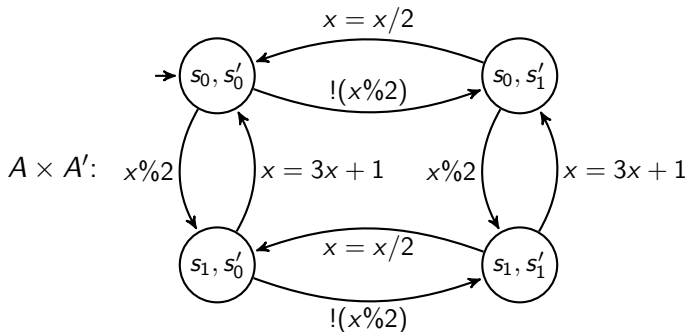
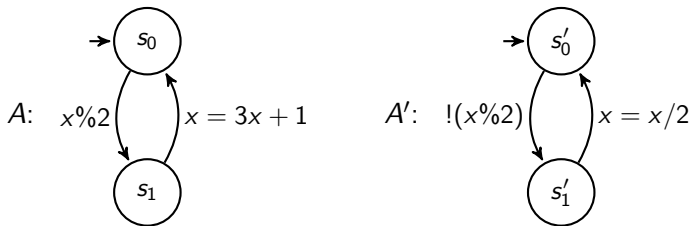
active proctype A1()
{
  do
    :: !(x%2) -> x = x/2
  od
}

```

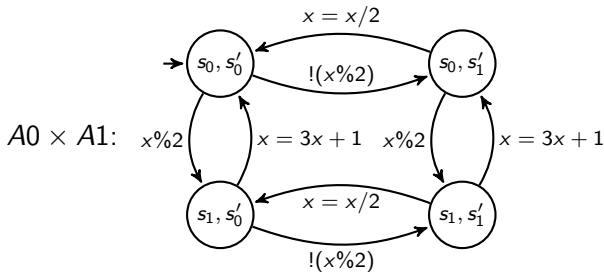


The transition labeled “ $x \% 2$ ” is enabled if $x \% 2 \neq 0$, i.e., if x is odd; “ $!(x \% 2)$ ” is enabled if x is even.

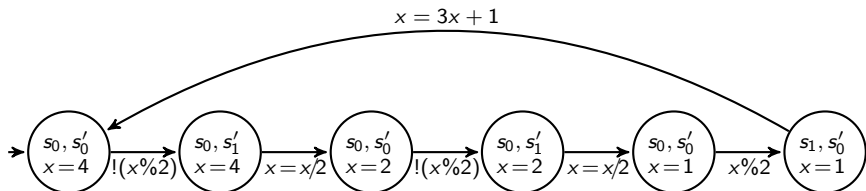
Interleaving as Asynchronous Product



Expanded Asynchronous Product



With $x = 4$ initially, we have a concrete finite-state automaton:



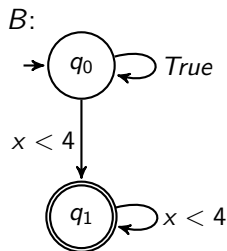
Specification as a Büchi Automaton

```

/* N was defined to be $4$ */
#define p (x < N)

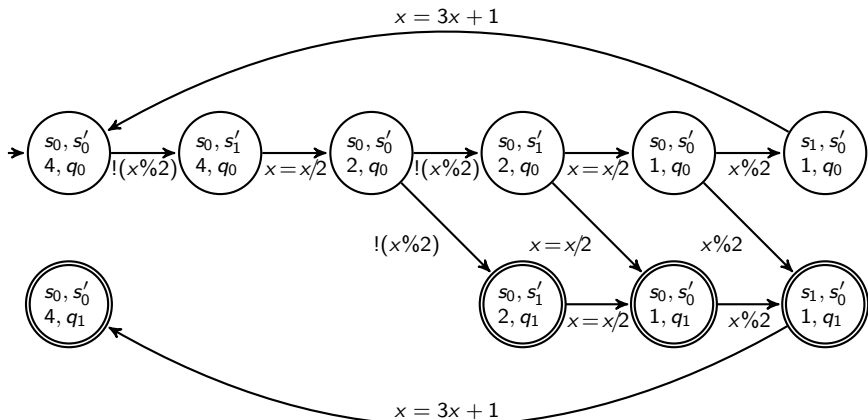
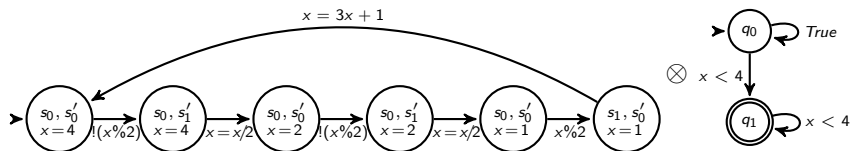
never { /* <>[]p */
T0_init:
  if
  :: p -> goto accept_S4
  :: true -> goto T0_init
  fi;
accept_S4:
  if
  :: p -> goto accept_S4
  fi;
}

```



Automaton *B* is equivalent to the “never claim”, which specifies all the bad behaviors.

Synchronous Product



On-the-Fly State Exploration

- 🌐 The automaton of the system under verification may be too large to fit into the memory.
- 🌐 Using the double DFS search for a counterexample, the system (the asynchronous product automaton) need not be expanded fully.
- 🌐 All we need to do are the following:
 - ☀️ Keep track of the current active search path.
 - ☀️ Compute the successor states of the current state.
 - ☀️ Remember (by hashing) states that have been visited.
- 🌐 This avoids construction of the entire system automaton and is referred to as *on-the-fly* state exploration.
- 🌐 The search can stop as soon as a counterexample is found.






Fairness

- 🌐 A concurrent system is composed of several concurrently executing processes.
- 🌐 Any process that can execute a statement should eventually proceed with that instruction, reflecting the very basic fact that a normal functioning processor has a positive speed.
- 🌐 This is the well-known notion of *weak fairness*, which is practically the most important kind of fairness.
- 🌐 Such fairness may be enforced in one of the following two ways:
 - ☀️ When searching for a counterexample, make sure that every process gets a chance to execute its next statement.
 - ☀️ Encode the fairness constraint in the specification automaton.

Concluding Remarks

- 🌐 Many techniques have been developed in the past to make the automata-based approach practical for real-world applications:
 - ☀ Partial order reduction
 - ☀ Abstraction refinement
 - ☀ Compositional reasoning
- 🌐 Most of these are still ongoing research.

References

-  J.R. Büchi. On a decision method in restricted second-order arithmetic, in *Proceedings of the 1960 International Congress on Logic, Methodology and Philosophy of Science*, Stanford University Press, 1962.
-  E.M. Clarke, O. Grumberg, and D.A. Peled. *Model Checking*, The MIT Press, 1999.
-  G.J. Holzmann. *The SPIN Model Checker: Primer and Reference Manual*, Addison-Wesley, 2003.
-  W. Thomas. Automata on infinite objects, *Handbook of Theoretical Computer Science* (Vol. B), 1990.
-  M.Y. Vardi and P. Wolper. An automata-theoretic approach to automatic program verification, in LICS 1986.