

Homework Assignment #2

Note

This assignment is due 9:10AM Wednesday, October 14, 2009. Please write or type your answers on A4 (or similar size) paper. Put your completed homework on the instructor's desk before the class starts. For late submissions, please drop them in Yih-Kuen Tsay's mail box on the first floor of Management Building II. A late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

We assume the binding powers of the logical connectives and the entailment symbol decrease in this order: \neg , $\{\forall, \exists\}$, $\{\wedge, \vee\}$, \rightarrow , \leftrightarrow , \vdash .

1. Prove, using Gentzen's System *LK*, the validity of the following sequents:

$$(a) \quad \forall x(P(x) \rightarrow Q(x)) \vdash \forall xP(x) \rightarrow \forall xQ(x) \quad (10 \text{ points})$$

$$(b) \quad \vdash \exists x\forall yP(x, y) \rightarrow \forall y\exists xP(x, y) \quad (10 \text{ points})$$

$$(c) \quad \exists xA(x) \rightarrow B \vdash \forall x(A(x) \rightarrow B), \text{ assuming } x \text{ does not occur free in } B. \quad (10 \text{ points})$$

2. Prove, using *Natural Deduction*, the validity of the following sequents:

$$(a) \quad \forall x(P(x) \rightarrow Q(x)) \vdash \forall xP(x) \rightarrow \forall xQ(x) \quad (10 \text{ points})$$

$$(b) \quad \vdash \exists x\forall yP(x, y) \rightarrow \forall y\exists xP(x, y) \quad (10 \text{ points})$$

$$(c) \quad \forall x(A(x) \rightarrow B) \vdash \exists xA(x) \rightarrow B, \text{ assuming } x \text{ does not occur free in } B. \quad (10 \text{ points})$$

3. Prove, using *Natural Deduction* for the first-order logic with equality ($=$), that $=$ is an equivalence relation between terms, i.e., the following are valid sequents, in addition to the obvious " $\vdash t = t$ " (Reflexivity), which follows from the $=$ -Introduction rule.

$$(a) \quad t_2 = t_1 \vdash t_1 = t_2 \text{ (Symmetry)} \quad (10 \text{ points})$$

$$(b) \quad t_1 = t_2, t_2 = t_3 \vdash t_1 = t_3 \text{ (Transitivity)} \quad (10 \text{ points})$$

4. Taking the preceding valid sequents as axioms, prove using *Natural Deduction* the following derived rules for equality.

$$(a) \quad \frac{\Gamma \vdash t_2 = t_1}{\Gamma \vdash t_1 = t_2} (= \textit{Symmetry}) \quad (10 \text{ points})$$

$$(b) \quad \frac{\Gamma \vdash t_1 = t_2 \quad \Gamma \vdash t_2 = t_3}{\Gamma \vdash t_1 = t_3} (= \textit{Transitivity}) \quad (10 \text{ points})$$