Inference Rules of Hoare Logic

$$\frac{}{\{Q[E/x]\}\ x := E\ \{Q\}} \qquad \text{(Assignment)}$$

Note: to treat multiple assignments, view x as a list of distinct variables and E as a list of expressions.

$$\frac{}{\{Q[(b; i : E)/b]\}\ b[i] := E\ \{Q\}} \qquad \text{(Assignment: array)}$$

$$\frac{}{\{P\}\ \mathbf{skip}\ \{P\}} \qquad \text{(Skip)}$$

$$\frac{\{P\}\ S_1\ \{Q\} \qquad \{Q\}\ S_2\ \{R\}}{\{P\}\ S_1; S_2\ \{R\}} \qquad \text{(Sequence)}$$

$$\frac{\{P \wedge B\}\ S_1\ \{Q\} \qquad \{P \wedge \neg B\}\ S_2\ \{Q\}}{\{P\}\ \mathbf{if}\ B\ \mathbf{then}\ S_1\ \mathbf{else}\ S_2\ \mathbf{fi}\ \{Q\}} \qquad \text{(Conditional)}$$

"**if** B **then** S **fi**" can be treated as "**if** B **then** S **else skip fi**" or directly with the following rule:

$$\frac{\{P \wedge B\}\ S\ \{Q\} \qquad P \wedge \neg B \to Q}{\{P\}\ \mathbf{if}\ B\ \mathbf{then}\ S\ \mathbf{fi}\ \{Q\}} \qquad \text{(If-Then)}$$

$$\frac{\{P \wedge B\}\ S\ \{P\}}{\{P\}\ \mathbf{while}\ B\ \mathbf{do}\ S\ \mathbf{od}\ \{P \wedge \neg B\}} \qquad (\mathbf{while})$$

$$\frac{P \to P' \qquad \{P'\}\ S\ \{Q'\} \qquad Q' \to Q}{\{P\}\ S\ \{Q\}} \qquad \text{(Consequence)}$$

$$\frac{\text{“}\mathbf{proc}\ \mathrm{p}(\mathbf{in}\ x;\ \mathbf{in\ out}\ y;\ \mathbf{out}\ z);\ \{P\}\ S\ \{Q\};\text{” is proved}}{\{P[a, b/x, y] \wedge I)\}\ \mathrm{p}(a, b, c)\ \{Q[b, c/y, z] \wedge I\}} \qquad \text{(Procedure Call)}$$

where b, c are (lists of) distinct variables and I does not refer to variables changed by procedure p.

$$\frac{\{P \wedge B\}\ S\ \{P\} \qquad \{P \wedge B \wedge t = Z\}\ S\ \{t < Z\} \qquad P \wedge B \to (t \geq 0)}{\{P\}\ \mathbf{while}\ B\ \mathbf{do}\ S\ \mathbf{od}\ \{P \wedge \neg B\}} \qquad (\mathbf{while}\text{: simply total})$$

$$\frac{\{P \wedge B\}\ S\ \{P\} \qquad \{P \wedge B \wedge \delta = D\}\ S\ \{\delta \prec D\} \qquad P \wedge B \to (\delta \in W)}{\{P\}\ \mathbf{while}\ B\ \mathbf{do}\ S\ \mathbf{od}\ \{P \wedge \neg B\}} \qquad (\mathbf{while}\text{: well-founded})$$

Auxiliary Rules:

$$\frac{P \to P' \qquad \{P'\}\ S\ \{Q\}}{\{P\}\ S\ \{Q\}} \qquad \text{(Strengthening Precondition)}$$

$$\frac{\{P\}\ S\ \{Q'\} \qquad Q' \to Q}{\{P\}\ S\ \{Q\}} \qquad \text{(Weakening Postcondition)}$$

$$\frac{\{P_1\}\ S\ \{Q_1\} \qquad \{P_2\}\ S\ \{Q_2\}}{\{P_1 \wedge P_2\}\ S\ \{Q_1 \wedge Q_2\}} \qquad \text{(Conjunction)}$$

$$\frac{\{P_1\}\ S\ \{Q_1\} \qquad \{P_2\}\ S\ \{Q_2\}}{\{P_1 \vee P_2\}\ S\ \{Q_1 \vee Q_2\}} \qquad \text{(Disjunction)}$$