Homework Assignment #1

Note

This assignment is due 2:20PM Thursday, September 29, 2011. Please write or type your answers on A4 (or similar size) paper. Put your completed homework on the instructor's desk before the class starts. For late submissions, please drop them in Yih-Kuen Tsay's mail box on the first floor of Management Building II. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

We assume the binding powers of the logical connectives and the entailment symbol decrease in this order: $\neg$, $\{\wedge, \vee\}$, $\rightarrow$, $\leftrightarrow$, $\vdash$.

1. Prove that every propositional formula has an equivalent formula in the conjunctive normal form and also an equivalent formula in the disjunctive normal form. (Hint: by induction on the structure of a formula, dealing with both cases simultaneously)   (30 points)

2. Prove, using *Natural Deduction* (in the sequent form), the validity of the following sequents:

    (a) $(p \rightarrow r) \wedge (q \rightarrow r) \vdash p \vee q \rightarrow r$ (20 points)

    (b) $\vdash (p \wedge q \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$ (20 points)

3. Prove, using *Natural Deduction* (in the sequent form), the validity of the following sequents:

    (a) $\vdash (p \rightarrow q) \rightarrow (\neg p \vee q)$ (15 points)

    (b) $\vdash ((p \rightarrow q) \rightarrow p) \rightarrow p$ (15 points)