Natural Deduction in the Sequent Form

(for Classic Logic)

$$\frac{}{\Gamma, A \vdash A}\ (Hyp)$$

$$\frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \wedge B}\ (\wedge I) \qquad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A}\ (\wedge E_1) \qquad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B}\ (\wedge E_2)$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \vee B}\ (\vee I_1) \qquad \frac{\Gamma \vdash B}{\Gamma \vdash A \vee B}\ (\vee I_2) \qquad \frac{\Gamma \vdash A \vee B \qquad \Gamma, A \vdash C \qquad \Gamma, B \vdash C}{\Gamma \vdash C}\ (\vee E)$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \to B}\ (\to I) \qquad \frac{\Gamma \vdash A \to B \qquad \Gamma \vdash A}{\Gamma \vdash B}\ (\to E)$$

$$\frac{\Gamma, A \vdash B \wedge \neg B}{\Gamma \vdash \neg A}\ (\neg I) \qquad \frac{\Gamma \vdash A \qquad \Gamma \vdash \neg A}{\Gamma \vdash B}\ (\neg E)$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash \neg\neg A}\ (\neg\neg I) \qquad \frac{\Gamma \vdash \neg\neg A}{\Gamma \vdash A}\ (\neg\neg E)$$

$$\frac{\Gamma \vdash A[y/x]}{\Gamma \vdash \forall x A}\ (\forall I) \qquad \frac{\Gamma \vdash \forall x A}{\Gamma \vdash A[t/x]}\ (\forall E)$$

$$\frac{\Gamma \vdash A[t/x]}{\Gamma \vdash \exists x A}\ (\exists I) \qquad \frac{\Gamma \vdash \exists x A \qquad \Gamma, A[y/x] \vdash B}{\Gamma \vdash B}\ (\exists E)$$

In the quantifier rules above, we assume that all substitutions are admissible and y does not occur free in Γ, A, or B.

Rules for Equality (an extension for languages with =):

Let t, t_1, t_2 be arbitrary terms and again assume all substitutions are admissible.

$$\frac{}{\Gamma \vdash t = t}\ (= I) \qquad \frac{\Gamma \vdash t_1 = t_2 \qquad \Gamma \vdash A[t_1/x]}{\Gamma \vdash A[t_2/x]}\ (= E)$$

Note: The = sign is part of the object language, not a meta symbol.