Homework Assignment #1

Due Time/Date

2:20PM Wednesday, September 21, 2022. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file. Name your file according to this pattern: "b097050xx-hw1'. Upload the PDF file to the NTU COOL site for Software Specification and Verification 2022. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

We assume the binding powers of the logical connectives and the entailment symbol decrease in this order: $\neg$, $\{\wedge, \vee\}$, $\rightarrow$, $\leftrightarrow$, $\vdash$.

1. (30 points) Prove that every propositional formula has an equivalent formula in the conjunctive normal form and also an equivalent formula in the disjunctive normal form. (Hint: by induction on the structure of a formula, dealing with both cases simultaneously)

2. (40 points) Prove, using *Natural Deduction* (in the sequent form), the validity of the following sequents:

   (a) $(p \rightarrow r) \wedge (q \rightarrow r) \vdash p \vee q \rightarrow r$

   (b) $\vdash (p \wedge q \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$

3. (30 points) Prove, using *Natural Deduction* (in the sequent form), the validity of the following sequents:

   (a) $\vdash (\neg p \vee q) \rightarrow (p \rightarrow q)$

   (b) $\vdash ((p \rightarrow q) \rightarrow p) \rightarrow p$