Homework Assignment #4

Due Time/Date

2:20PM Wednesday, November 1, 2023. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please write or type your answers on A4 (or similar size) paper. Put your completed homework on the instructor's desk before the class starts. For late submissions, please drop them in Yih-Kuen Tsay's mail box on the first floor of Management Building 2. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

We assume the binding powers of the logical connectives and the entailment symbol decrease in this order: $\neg$, $\{\forall, \exists\}$, $\{\wedge, \vee\}$, $\rightarrow$, $\leftrightarrow$, $\vdash$.

1. Prove that the following annotated program segments are correct:

   (a) (10 points)

   $\{true\}$
   if $x < y$ **then** $x, y := y, x$ **fi**
   $\{x \geq y\}$

   (b) (10 points)

   $\{g = 0 \wedge p = n \wedge n \geq 1\}$
   while $p \geq 2$ **do**
   $\quad g, p := g + 1, p - 1$
   od
   $\{g = n - 1\}$

   (c) (20 points) For this program, prove its total correctness.

   $\{y > 0 \wedge (x \equiv m \pmod{y})\}$
   while $x \geq y$ **do**
   $\quad x := x - y$
   od
   $\{(x \equiv m \pmod{y}) \wedge x < y\}$

2. (30 %) Below is a program that finds the minimum and the maximum elements of an array of n (assumed to be positive and even) integers. The elements of an array are indexed from 1 through n.

```
if (a[1] < a[2]) then
  min := a[1];
  max := a[2]
else
  min := a[2];
  max := a[1]
fi;

i := 3;
while (i<=n) do
  if (a[i] < a[i+1]) then
    if (a[i] < min) then
      min := a[i]
    fi;
    if (a[i+1] > max) then
      max := a[i+1];
    fi
  else
    if (a[i+1] < min) then
      min := a[i+1]
    fi;
    if (a[i] > max) then
      max := a[i]
    fi
  fi;
  i := i + 2;
od;
```

Annotate the program into a *standard* proof outline, showing clearly the partial correctness of the program; a standard proof outline is essentially an annotated program where every statement is preceded by a pre-condition and the entire program is followed by a post-condition.

3. (30 points) Given a sequence $x_1, x_2, \cdots, x_n$ of real numbers (not necessarily positive), a maximum subsequence $x_i, x_{i+1}, \cdots, x_j$ is a subsequence of consecutive elements from the given sequence such that the sum of the numbers in the subsequence is maximum over all subsequences of consecutive elements. Below is a program that determines the sum of such a sequence.

```
Global_Max := 0;
Suffix_Max := 0;
i := 1;
while (i<=n) do
   if x[i] + Suffix_Max > Global_Max then
      Suffix_Max := Suffix_Max + x[i];
      Global_Max := Suffix_Max
   else
     if x[i] + Suffix_Max > 0 then
       Suffix_Max := Suffix_Max + x[i]
     else Suffix_Max := 0
     fi
   fi;
   i := i + 1
 od;
```

Annotate the program into a standard proof outline, showing clearly the partial correctness of the program.