Homework Assignment #8

Note

This assignment is due 2:10PM Wednesday, May 15, 2013. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II, or put it on the instructor's desk before the class on the due date starts. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2006] with probable adaptation.)

1. (10 points) Give a formal definition of a Turing machine that copies and appends the input string to the end of the original input with an additional # to separate the two strings. The input alphabet is $\{0, 1\}$.

2. (Exercise 3.4; 10 points) Give a formal definition of an enumerator. Consider it to be a type of two-tape Turing machine that uses its second tape as the printer. Include a definition of the enumerated language.

3. (Problem 3.18; 20 points) Show that a language is decidable iff some enumerator enumerates the language in lexicographical order.

4. (Problem 3.20; 20 points) Show that single-tape TMs that cannot write on the portion of the tape containing the input string recognize only regular languages.

5. (Problem 3.21; 20 points) Let $c_1x^n + c_2x^{n-1} + \cdots + c_nx + c_{n+1}$ be a polynomial with a root at $x = x_0$. Let $c_{\max}$ be the largest absolute value of a c_i. Show that

$$|x_0| < (n+1)\frac{c_{\max}}{|c_1|}.$$

6. (Exercise 4.3; 10 points) Let $ALL_{\text{DFA}} = \{\langle A\rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$. Show that ALL_{DFA} is decidable.

7. (Problem 4.12; 10 points) Let $A = \{\langle R, S\rangle \mid R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S)\}$. Show that A is decidable.