Homework Assignment #2

Note

This assignment is due 2:10PM Wednesday, March 12, 2014. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II, or put it on the instructor's desk before the class on the due date starts. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Exercise 1.3; 10 points) The formal definition of a DFA M is $(\{q_1, q_2, q_3, q_4, q_5\}, \{\texttt{u}, \texttt{d}\}, \delta, q_3, \{q_3\})$ where δ is given by the following table. Draw the state diagram of this machine.

	u	d
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_5

2. (Exercise 1.4; 20 points) Each of the following languages is the intersection of two simpler regular languages. In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in class (see also Footnote 3 in Page 46 of [Sipser 2006]) to give the state diagram of a DFA for the language given. In all parts, the alphabet is $\{\texttt{a}, \texttt{b}\}$.

   (a) $\{w \mid w$ starts with an a and has at most one b$\}$.

   (b) $\{w \mid w$ has an odd number of a's and ends with a b$\}$.

3. (Exercise 1.5; 20 points) Each of the following languages is the complement of a simpler regular language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts, the alphabet is $\{\texttt{a}, \texttt{b}\}$.

   (a) $\{w \mid w$ contains neither the substring ab nor ba$\}$.

   (b) $\{w \mid w$ is any string except a and b$\}$.

4. (Exercise 1.6; 20 points) Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is $\{\texttt{0}, \texttt{1}\}$.

   (a) $\{w \mid w$ contains the substring 0101, i.e., $w = x\texttt{0101}y$ for some x and $y\}$.

(b) $\{w \mid \text{every odd position of } w \text{ is a } \mathtt{1}\}$ (Note: see w as $w_1w_2\cdots w_n$, where $w_i \in \{\mathtt{0},\mathtt{1}\}$).

5. (Problem 1.36; 10 points) For any string $w = w_1w_2\cdots w_n$, the *reverse* of w, written w^R, is the string w in reverse order, $w_n \cdots w_2w_1$. For any language A, let $A^R = \{w^R \mid w \in A\}$. Show that if A is regular, so is A^R.

6. (Problem 1.37; 10 points) Let

$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

Σ_3 contains all size 3 columns of 0s and 1s. A string of symbols in Σ_3 gives three rows of 0s and 1s. Consider each row to be a binary number and let

$$B = \{w \in \Sigma_3 \mid \text{the bottom row of } w \text{ is the sum of the top two rows}\}.$$

For example,

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in B, \text{but} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \notin B.$$

Show that B is regular. (Hint: working with B^R is easier. You may assume the result claimed in the previous problem (Problem 1.36).)

7. (10 points) Generalize the proof of Theorem 1.25 of [Sipser 2006, 2013] (Pages 45–47) to handle A_1 and A_2 with different alphabets.