Homework Assignment #6

Note

This assignment is due 2:10PM Wednesday, April 23, 2014. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II, or put it on the instructor's desk before the class on the due date starts. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2006] with probable adaptation.)

1. (Exercise 2.2; 20 points)

    (a) Use the languages $A = \{a^m b^n c^n \mid m, n \geq 0\}$ and $B = \{a^n b^n c^m \mid m, n \geq 0\}$, together with the fact that $\{a^n b^n c^n \mid m, n \geq 0\}$ is not context free, to show that the class of context-free languages is not closed under intersection.

    (b) Use the preceding part and DeMorgan's law to show that the class of context-free languages is not closed under complementation.

2. (Exercise 2.5; 20 points) Give informal descriptions and state diagrams of pushdown automata for the following languages. In all parts the alphabet Σ is $\{0, 1\}$.

    (a) $\{w \mid \text{the length of } w \text{ is even}\}$

    (b) $\{w \mid w = w^R, \text{ that is, } w \text{ is a palindrome}\}$

3. (Exercise 2.12; 10 points) Convert the following CFG to an equivalent PDA, using the procedure given in Theorem 2.20.

$$
\begin{aligned}
E &\rightarrow E + T \mid T \\
T &\rightarrow T \times F \mid F \\
F &\rightarrow (E) \mid a
\end{aligned}
$$

4. (Problem 2.32; 10 points) Let $A/B = \{w \mid wx \in A \text{ for some } x \in B\}$. Show that, if A is context free and B is regular, then A/B is context free.

5. (Problem 2.39; 20 points) Let $G = (V, \Sigma, R, \langle\text{STMT}\rangle)$ be the following grammar.

$$
\begin{aligned}
\langle\text{STMT}\rangle &\rightarrow \langle\text{ASSIGN}\rangle \mid \langle\text{IF-THEN}\rangle \mid \langle\text{IF-THEN-ELSE}\rangle \\
\langle\text{IF-THEN}\rangle &\rightarrow \texttt{if condition then } \langle\text{STMT}\rangle \\
\langle\text{IF-THEN-ELSE}\rangle &\rightarrow \texttt{if condition then } \langle\text{STMT}\rangle \texttt{ else } \langle\text{STMT}\rangle \\
\langle\text{ASSIG}\rangle &\rightarrow \texttt{a := 1}
\end{aligned}
$$

$$\Sigma = \{\texttt{if}, \texttt{condition}, \texttt{then}, \texttt{else}, \texttt{a := 1}\}$$

$$V = \{\langle\text{STMT}\rangle, \langle\text{IF-THEN}\rangle, \langle\text{IF-THEN-ELSE}\rangle, \langle\text{ASSIG}\rangle\}$$

G is a *natural-looking* grammar for a fragment of a programming language, but G is ambiguous.

(a) Show that G is ambiguous.

(b) Give a new unambiguous grammar for the same language.

6. (Problem 2.56; 10 points) If A and B are languages, define $A \diamond B = \{xy \mid x \in A \text{ and } y \in B \text{ and } |x| = |y|\}$. Show that if A and B are regular, then $A \diamond B$ is context free.

7. (Problem 2.42; 10 points) Use the pumping lemma to show that the language $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$ is not context free.