

## Homework Assignment #3

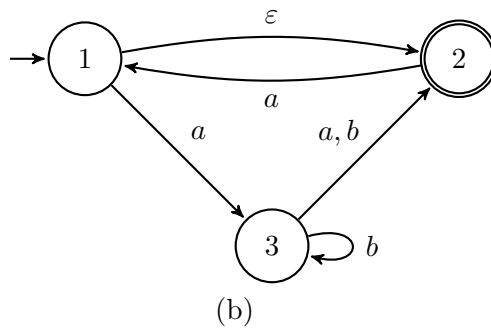
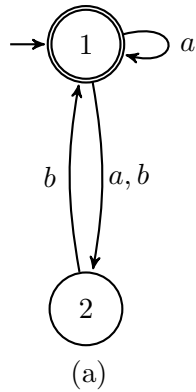
### Note

This assignment is due 2:10PM Wednesday, March 25, 2015. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II, or put it on the instructor's desk before the class on the due date starts. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

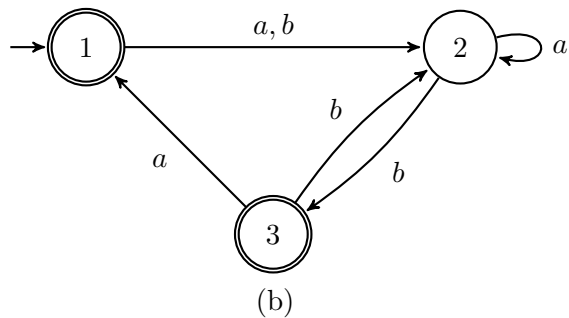
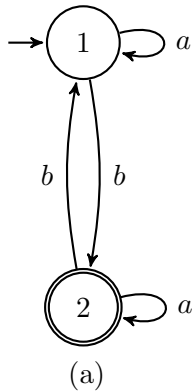
### Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

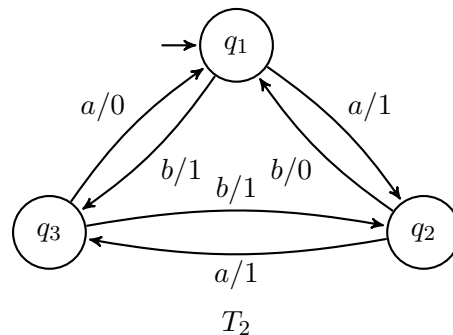
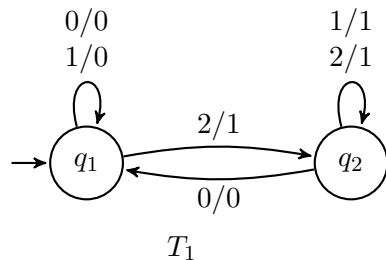
1. (Exercise 1.8; 10 points) Use the construction given in the proof (using NFAs) of Theorem 1.45 (closedness of regular languages under union) to give the state diagram of an NFA recognizing the union of  $\{w \mid w \text{ begins with a } 1 \text{ and ends with a } 0\}$  and  $\{w \mid w \text{ contains at least three } 1\text{s}\}$ . The alphabet is  $\{0, 1\}$ .
2. (Exercise 1.10; 10 points) Use the construction given in the proof (using NFAs) of Theorem 1.49 (closedness of regular languages under the Kleene star) to give the state diagram of an NFA recognizing the star of  $\{w \mid w \text{ contains at least three } 1\text{s}\}$ . The alphabet is  $\{0, 1\}$ .
3. (Exercise 1.12; 10 points) Let  $D = \{w \mid w \text{ contains an even number of } \mathbf{a}\text{'s and an odd number of } \mathbf{b}\text{'s and does not contain the substring } \mathbf{ab}\}$ . Give a DFA with five states that recognizes  $D$  and a regular expression that generates  $D$ . (Suggestion: describe  $D$  in a simpler way.)
4. (Exercise 1.14; 10 points) Show by giving an example that, if  $M$  is an NFA that recognizes language  $C$ , swapping the accept and nonaccept states in  $M$  doesn't necessarily yield a new NFA that recognizes the complement of  $C$ . Is the class of languages recognized by NFAs closed under complement? Explain your answer.
5. (Exercise 1.16; 10 points) Use the construction given in Theorem 1.39 (every NFA has an equivalent DFA) to convert the following two nondeterministic finite automata to equivalent deterministic finite automata.



6. (Exercise 1.18; 10 points) Use the procedure described in Lemma 1.55 to convert the regular expression  $(0 \cup 1)^*000(0 \cup 1)^*$  to a nondeterministic finite automaton.
7. (Exercise 1.20; 10 points) Give regular expressions generating the following languages:
  - (a)  $\{w \mid w \text{ contains the substring } 0101, \text{ i.e., } w = x0101y \text{ for some } x \text{ and } y\}$
  - (b)  $\{w \mid w \text{ doesn't contain the substring } 110\}$
8. (Exercise 1.21; 10 points) Use the procedure described in Lemma 1.60 to convert the following finite automata to regular expressions.



9. (Exercise 1.24; 10 points) A *finite-state transducer* (FST) is a type of deterministic finite automaton whose output is a string rather than *accept* or *reject*. The following are state diagrams of finite state transducers  $T_1$  and  $T_2$ .



Each transition of an FST is labeled with two symbols, one designating the input symbol for that transition and the other designating the output symbol. The two symbols are written with a slash, /, separating them. In  $T_1$ , the transition from  $q_1$  to  $q_2$  has input symbol 2 and output symbol 1. Some conditions may have multiple input-output pairs, such as the transition in  $T_1$  from  $q_1$  to itself. When an FST computes on an input string  $w$ , it takes the input symbols  $w_1 \cdots w_n$  one by one and, starting from the start state, follows the transitions by matching the input labels with the sequence of symbols  $w_1 \cdots w_n = w$ . Every time it goes along a transition, it outputs the corresponding output symbol. For example, on input 2212011, machine  $T_1$  enters the sequence of states  $q_1, q_2, q_2, q_2, q_2, q_1, q_1, q_1$  and produces output 1111000. On input abbb,  $T_2$  outputs 1011. Give the sequence of states entered and the output produced in each of the following parts.

- (a)  $T_1$  on input 12021
  - (b)  $T_2$  on input bbabba
10. (Exercise 1.25; 10 points) Read the informal definition of the finite state transducer given in Exercise 1.24. Give a formal definition of this model, following the patterns in Definition 1.5 (Page 35). Assume that an FST has an input alphabet  $\Sigma$  and an output alphabet  $\Gamma$  but not a set of accept states. Include a formal definition of the computation of an FST. (Hint: an FST is a 5-tuple. Its transition function is of the form  $\delta : Q \times \Sigma \rightarrow Q \times \Gamma$ .)