Homework Assignment #7

Note

This assignment is due 2:10PM Wednesday, May 11, 2016. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building II, or put it on the instructor's desk before the class on the due date starts. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (20 points) Give a *formal* description of a Turing machine that decides the language $\{w \mid w$ contains an equal number of 1s and 0s$\}$.

2. (10 points) Give a *formal* description of a Turing machine that decides the language $\{w \mid w$ contains twice as many 1s as 0s$\}$.

3. (Exercise 3.7; 10 points) Explain why the following is not a description of a legitimate Turing machine.

   M_{bad} = "The input is a polynomial p over variables $x_1, \ldots, x_k$:

   (a) Try all possible settings of $x_1, \ldots, x_k$ to integer values.
   (b) Evaluate p on all of these settings.
   (c) If any of these settings evaluates to 0, *accept*; otherwise, *reject*."

4. (Problem 3.16; 10 points) Show that the collection of decidable languages is closed under *concatenation*.

5. (Problem 3.19; 10 points) A ***Turing machine with left reset*** is similar to an ordinary Turing machine, but the transition function has the form

   $$\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, RESET\}$$

   If $\delta(q, a) = (r, b, RESET)$, when the machine is in state q reading an a, the machine's head jumps to the left-hand end of the tape after it writes b on the tape and enters state r. Note that these machines do not have the usual ability to move the head one symbol left. Show that Turing machines with left reset recognize the class of Turing-recognizable languages.

6. (Problem 3.20; 20 points) A ***Turing machine with stay put instead of left*** is similar to an ordinary Turing machine, but the transition function has the form

   $$\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, S\}$$

   At each point the machine can move instead its head right or let it stay in the same position. Show that this Turing machine variant is *not* equivalent to the usual version. What class of languages do these machines recognize?

7. (Problem 3.22; 20 points) Let a k-PDA be a pushdown automaton that has k stacks. Thus a 0-PDA is an NFA and a 1-PDA is a conventional PDA. You already know that 1-PDAs are more powerful (recognizing a larger class of languages) than 0-PDAs.

   (a) Show that 2-PDAs are more powerful than 1-PDAs.

   (b) Show that 3-PDAs are *not* more powerful than 2-PDAs. (Hint: simulate a Turing machine tape with two stacks.)