# Homework Assignment #4

## Due Time/Date

This assignment is due 2:10PM Tuesday, April 7, 2020. Late submission will be penalized by 20% for each working day overdue.

## How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file. Name your file according to this pattern: "b057050xx-hw4". Upload the PDF file to the Ceiba course site for Theory of Computing 2020: https://ceiba.ntu.edu.tw/1082theory2020. You may discuss the problems with others, but copying answers is strictly forbidden.

## Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Problem 1.32; 20 points) For languages $A$ and $B$, let the *shuffle* of $A$ and $B$ be the language $\{w \mid w = a_1 b_1 \cdots a_k b_k$, where $a_1 \cdots a_k \in A$ and $b_1 \cdots b_k \in B$, each $a_i, b_i \in \Sigma^*\}$. Show that the class of regular languages is closed under shuffle.

2. (Problem 1.38; 20 points) Let

$$\Sigma_2 = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}.$$

   Here, $\Sigma_2$ contains all columns of 0s and 1s of length two. A string of symbols in $\Sigma_2$ gives two rows of 0s and 1s. Consider each row to be a binary number and let

$$C = \{w \in \Sigma_2^* \mid \text{the bottom row of } w \text{ is three times the top row}\}.$$

   For example, $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \in C$, but $\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \notin C$. Show that $C$ is regular. (You may assume the result claimed in Problem 5 of HW#2.)

3. (Problem 1.40; 10 points) Let $\Sigma_2$ be the same as in Problem 2. Consider the top and bottom rows to be strings of 0s and 1s and let

$$E = \{w \in \Sigma_2^* \mid \text{the bottom row of } w \text{ is the reverse of the top row of } w\}.$$

   Show that $E$ is not regular.

4. (Problem 1.42; 20 points) Let $C_n = \{x \mid x \text{ is a binary number that is a multiple of } n\}$. Show that for each $n \geq 1$, the language $C_n$ is regular.

5. (Problem 1.51; 10 points) Prove that the language $\{w \in \{0,1\}^* \mid w \text{ is not a palindrome}\}$ is not regular. You may use the pumping lemma and the closedness of the class of regular languages under union, intersection, and complement. (Note: a *palindrome* is a string that reads the same forward and backward.)

6. (Problem 1.66; 20 points) Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA and let $h$ be a state of $M$ called its "home". A *synchronizing sequence* for $M$ and $h$ is a string $s \in \Sigma^*$ where $\delta(q, s) = h$ for every $q \in Q$. Say that $M$ is *synchronizable* if it has a synchronizing sequence for some state $h$. Prove that, if $M$ is a $k$-state synchronizable DFA, then it has a synchronizing sequence of length at most $k^3$. (Note: $\delta(q, s)$ equals the state where $M$ ends up, when $M$ starts from state $q$ and reads input $s$.)