

Homework Assignment #6

Due Time/Date

This assignment is due 2:10PM Tuesday, April 28, 2020; however, to be better prepared for the midterm exam on April 21, you should try to complete it before the exam. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file. Name your file according to this pattern: “b057050xx-hw6”. Upload the PDF file to the Ceiba course site for Theory of Computing 2020: <https://ceiba.ntu.edu.tw/1082theory2020>. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with “Exercise X.XX” or “Problem X.XX” are taken from [Sipser 2006, 2013] with probable adaptation.)

1. (Exercise 2.2; 20 points)
 - (a) Use the languages $A = \{a^n b^n c^m \mid m, n \geq 0\}$ and $B = \{a^m b^n c^n \mid m, n \geq 0\}$, together with the fact that $\{a^n b^n c^n \mid m, n \geq 0\}$ is not context free, to show that the class of context-free languages is not closed under intersection.
 - (b) Use the preceding part and DeMorgan’s law to show that the class of context-free languages is not closed under complementation.
2. (Exercise 2.5; 20 points) Give informal descriptions and state diagrams of pushdown automata for the following languages. In all parts the alphabet Σ is $\{0, 1\}$.
 - (a) $\{w \mid \text{the length of } w \text{ is odd}\}$
 - (b) $\{w \mid w = w^R, \text{ that is, } w \text{ is a palindrome}\}$
3. (Exercise 2.12; 20 points) Convert the following CFG to an equivalent PDA, using the procedure given in Theorem 2.20.

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T \times F \mid F \\ F &\rightarrow (E) \mid a \end{aligned}$$

4. (Problem 2.32; 10 points) Let $A/B = \{w \mid wx \in A \text{ for some } x \in B\}$. Show that, if A is context free and B is regular, then A/B is context free.
5. (Problem 2.39; 20 points) Let $G = (V, \Sigma, R, \langle \text{STMT} \rangle)$ be the following grammar.

$$\begin{aligned} \langle \text{STMT} \rangle &\rightarrow \langle \text{ASSIGN} \rangle \mid \langle \text{IF-THEN} \rangle \mid \langle \text{IF-THEN-ELSE} \rangle \\ \langle \text{IF-THEN} \rangle &\rightarrow \text{if condition then } \langle \text{STMT} \rangle \\ \langle \text{IF-THEN-ELSE} \rangle &\rightarrow \text{if condition then } \langle \text{STMT} \rangle \text{ else } \langle \text{STMT} \rangle \\ \langle \text{ASSIG} \rangle &\rightarrow \text{a} := 1 \end{aligned}$$

$$\Sigma = \{\text{if, condition, then, else, a := 1}\}$$

$$V = \{\langle \text{STMT} \rangle, \langle \text{IF-THEN} \rangle, \langle \text{IF-THEN-ELSE} \rangle, \langle \text{ASSIG} \rangle\}$$

G is a *natural-looking* grammar for a fragment of a programming language, but G is ambiguous.

- (a) Show that G is ambiguous.
 - (b) Give a new unambiguous grammar for the same language.
6. (Problem 2.42; 10 points) Use the pumping lemma to show that the language $\{1^n 0^n 1^n 0^n \mid n \geq 0\}$ is not context free.