

Homework Assignment #10

Due Time/Date

This assignment is due 2:10PM Tuesday, June 8, 2021. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file. Name your file according to this pattern: “b067050xx-hw10”. Upload the PDF file to the NTU COOL course site for Theory of Computing 2021. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with “Exercise X.XX” or “Problem X.XX” are taken from [Sipser 2013] with probable adaptation.)

1. (Problem 5.9; 10 points) Let $AMBIG_{CFG} = \{\langle G \rangle \mid G \text{ is an ambiguous CFG}\}$. Show that $AMBIG_{CFG}$ is undecidable. (Hint: use a reduction from PCP. Given an instance

$$P = \left\{ \left[\frac{t_1}{b_1} \right], \left[\frac{t_2}{b_2} \right], \dots, \left[\frac{t_k}{b_k} \right] \right\}$$

of PCP, construct a CFG G with the rules:

$$\begin{aligned} S &\rightarrow T \mid B \\ T &\rightarrow t_1 T a_1 \mid \dots \mid t_k T a_k \mid t_1 a_1 \mid \dots \mid t_k a_k \\ B &\rightarrow t_1 B a_1 \mid \dots \mid t_k B a_k \mid t_1 a_1 \mid \dots \mid t_k a_k, \end{aligned}$$

where a_1, \dots, a_k are new terminal symbols. Prove that this reduction works.)

2. (Problem 5.14(b); 20 points) Define a *two-headed finite automaton* (2DFA) to be a deterministic finite automaton that has two read-only, bidirectional heads that start at the left-hand end of the input tape and can be independently controlled to move in either direction. The tape of a 2DFA is finite and is just large enough to contain the input plus two additional blank tape cells, one on the left-end and one on the right-hand end, that serve as delimiters. A 2DFA accepts its input by entering a special accept state. For example, a 2DFA can recognize the language $\{a^n b^n c^n \mid n \geq 0\}$.

Let $E_{2DFA} = \{\langle M \rangle \mid M \text{ is a 2DFA and } L(M) = \emptyset\}$. Show that E_{2DFA} is undecidable.

3. (Problem 5.18(c); 10 points) Use Rice’s theorem to prove the undecidability of the language $\{\langle M \rangle \mid M \text{ is a TM and } L(M) = \Sigma^*\}$. (Note: you should show that Rice’s theorem is applicable for the problem/language.)
4. (Problem 5.22; 20 points) Let $X = \{\langle M, w \rangle \mid M \text{ is a single-tape TM that never modifies the portion of the tape that contains the input } w\}$. Is X decidable? Prove your answer.

5. (20 points) Prove that $HALT_{TM} \leq_m \overline{E_{TM}}$, where $HALT_{TM} = \{\langle M, w \rangle \mid M \text{ is a TM and } M \text{ halts on } w\}$ and $E_{TM} = \{\langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset\}$.
6. (10 points) Let $ALL_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$. Prove that $ALL_{DFA} \in P$.
7. (10 points) Two graphs G and H are said to be *isomorphic* if the nodes of G may be re-named so that it becomes identical to H . Let $ISO = \{\langle G, H \rangle \mid G \text{ and } H \text{ are isomorphic}\}$. Prove that $ISO \in NP$, using the definition $NP = \bigcup_k NTIME(n^k)$.