Homework Assignment #10

Due Time/Date

This assignment is due 1:20PM Tuesday, May 31, 2022. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file. Name your file according to this pattern: "b077050xx-hw10". Upload the PDF file to the NTU COOL course site for Theory of Computing 2021. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Problem 5.9; 10 points) Let $AMBIG_{\mathrm{CFG}} = \{\langle G\rangle \mid G \text{ is an ambiguous CFG}\}$. Show that $AMBIG_{\mathrm{CFG}}$ is undecidable. (Hint: use a reduction from PCP. Given an instance

$$P = \left\{ \left[\frac{t_1}{b_1}\right], \left[\frac{t_2}{b_2}\right], \ldots, \left[\frac{t_k}{b_k}\right] \right\}$$

of PCP, construct a CFG G with the rules:

$$\begin{array}{lcl} S & \to & T \mid B \\ T & \to & t_1 T a_1 \mid \cdots \mid t_k T a_k \mid t_1 a_1 \mid \cdots \mid t_k a_k \\ B & \to & b_1 B a_1 \mid \cdots \mid b_k B a_k \mid b_1 a_1 \mid \cdots \mid b_k a_k, \end{array}$$

where $a_1, \ldots, a_k$ are new terminal symbols. Prove that this reduction works.)

2. (Problem 5.14(b); 20 points) Define a *two-headed finite automaton* (2DFA) to be a deterministic finite automaton that has two read-only, bidirectional heads that start at the left-hand end of the input tape and can be independently controlled to move in either direction. The tape of a 2DFA is finite and is just large enough to contain the input plus two additional blank tape cells, one on the left-end and one on the right-hand end, that serve as delimiters. A 2DFA accepts its input by entering a special accept state. For example, a 2DFA can recognize the language $\{a^n b^n c^n \mid n \geq 0\}$.

   Let $E_{\mathrm{2DFA}} = \{\langle M\rangle \mid M \text{ is a 2DFA and } L(M) = \emptyset\}$. Show that E_{2DFA} is undecidable.

3. (Problem 5.18(b); 10 points) Use Rice's theorem to prove the undecidability of the language $\{\langle M\rangle \mid M \text{ is a TM and } 101 \in L(M)\}$. (Note: you should show that Rice's theorem is applicable for the problem/language.)

4. (Problem 5.22; 20 points) Let $X = \{\langle M, w\rangle \mid M \text{ is a single-tape TM that never modifies the portion of the tape that contains the input } w\}$. Is X decidable? Prove your answer.

5. (20 points) Prove that $HALT_{\mathrm{TM}} \leq_m \overline{E_{\mathrm{TM}}}$, where $HALT_{\mathrm{TM}} = \{\langle M, w\rangle \mid M$ is a TM and M halts on $w\}$ and $E_{\mathrm{TM}} = \{\langle M\rangle \mid M$ is a TM and $L(M) = \emptyset\}$.

6. (10 points) Let $ALL_{\mathrm{DFA}} = \{\langle A\rangle \mid A$ is a DFA and $L(A) = \Sigma^*\}$. Prove that $ALL_{\mathrm{DFA}} \in \mathrm{P}$.

7. (10 points) Two graphs G and H are said to be *isomorphic* if the nodes of G may be renamed so that it becomes identical to H. Let $ISO = \{\langle G, H\rangle \mid G$ and H are isomorphic$\}$. Prove that $ISO \in \mathrm{NP}$, using the definition $\mathrm{NP} = \bigcup_k \mathrm{NTIME}(n^k)$.