

Homework Assignment #9

Due Time/Date

This assignment is due 2:20PM Tuesday, May 24, 2022. Late submission will be penalized by 20% for each working day overdue.

How to Submit

Please use a word processor or scan hand-written answers to produce a single PDF file. Name your file according to this pattern: “b077050xx-hw9”. Upload the PDF file to the NTU COOL course site for Theory of Computing 2022. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with “Exercise X.XX” or “Problem X.XX” are taken from [Sipser 2013] with probable adaptation.)

1. (Exercise 4.9; 10 points) Review the way that we define sets to be of the same size in Definition 4.12. Show that “are of the same size” is an equivalence relation.
2. (Problem 4.12; 10 points) Let A be a Turing-recognizable language consisting of descriptions of Turing machines, $\{\langle M_1 \rangle, \langle M_2 \rangle, \dots\}$, where every M_i is a decider. Prove that some decidable language D is not decided by any decider M_i whose description appears in A . (Hint: you may find it helpful to consider an enumerator for A .)
3. (Problem 4.14; 20 points) Let $C = \{\langle G, x \rangle \mid G \text{ is a CFG and } x \text{ is a substring of some } y \in L(G)\}$. Show that C is decidable. (Hint: an elegant solution to this problem uses the decider for E_{CFG} .)
4. (Problem 4.18; 20 points) A *useless state* in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.
5. (Problem 4.31; 20 points) Let $INFINITE_{\text{PDA}} = \{\langle M \rangle \mid M \text{ is a PDA and } L(M) \text{ is infinite}\}$. Show that $INFINITE_{\text{PDA}}$ is decidable.
6. (Exercise 5.4; 20 points) If A is reducible to B and B is a regular language, does that imply that A is a regular language? Why or why not?