Homework Assignment #10

Due Time/Date

This assignment is due 1:20PM Tuesday, May 30, 2023. Late submission will be penalized by 20% for each working day overdue.

Note

Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building 2, or put it on the instructor's desk before the class (or the TA session) on the due date starts. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Problem 5.12; 10 points) Let $J = \{w \mid \text{either } w = 0x \text{ for some } x \in A_{\text{TM}}, \text{ or } w = 1y \text{ for some } y \in \overline{A_{\text{TM}}}\}$. Show that neither J nor $\overline{J}$ is Turing-recognizable.

2. (Problem 5.14(b); 20 points) Define a *two-headed finite automaton* (2DFA) to be a deterministic finite automaton that has two read-only, bidirectional heads that start at the left-hand end of the input tape and can be independently controlled to move in either direction. The tape of a 2DFA is finite and is just large enough to contain the input plus two additional blank tape cells, one on the left-end and one on the right-hand end, that serve as delimiters. A 2DFA accepts its input by entering a special accept state. For example, a 2DFA can recognize the language $\{a^n b^n c^n \mid n \geq 0\}$.

   Let $E_{\text{2DFA}} = \{\langle M \rangle \mid M \text{ is a 2DFA and } L(M) = \emptyset\}$. Show that E_{2DFA} is undecidable.

3. (Problem 5.18 adapted; 20 points) Please discuss briefly the applicability of Rice's theorem to proving the undecidability of each of the following languages.

   (a) $HALT_{\text{TM}} = \{\langle M, w \rangle \mid M \text{ is a TM and } M \text{ halts on } w\}$.

   (b) $REGULAR_{\text{TM}} = \{\langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is regular}\}$.

   (c) $E_{\text{LBA}} = \{\langle M \rangle \mid M \text{ is an LBA where } L(M) = \emptyset\}$.

   (d) $ALL_{\text{CFG}} = \{\langle G \rangle \mid G \text{ is a CFG and } L(G) = \Sigma^*\}$.

4. (Problem 5.22; 20 points) Let $X = \{\langle M, w \rangle \mid M \text{ is a single-tape TM that never modifies the portion of the tape that contains the input } w\}$. Is X decidable? Prove your answer.

5. (Problem 5.23(b); 10 points) A variable A in CFG G is said to be *necessary* if it appears in every derivation of some string $w \in L(G)$. Let $NECESSARY_{\text{CFG}} = \{\langle G, A \rangle \mid A \text{ is a necessary variable in } G\}$. Prove that $NECESSARY_{\text{CFG}}$ is undecidable.

6. (10 points) Let $ALL_{\text{DFA}} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$. Prove that $ALL_{\text{DFA}} \in \text{P}$.

7. (10 points) Two graphs G and H are said to be *isomorphic* if the nodes of G may be renamed so that it becomes identical to H. Let $ISO = \{\langle G, H\rangle \mid G \text{ and } H \text{ are isomorphic}\}$. Prove that $ISO \in \text{NP}$, using the definition $\text{NP} = \bigcup_k \text{NTIME}(n^k)$.