# Homework Assignment #2

## Due Time/Date

This assignment is due 1:20PM Tuesday, March 21, 2023. Late submission will be penalized by 20% for each working day overdue.

## Note

Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building 2, or put it on the instructor's desk before the class on the due date starts. You may discuss the problems with others, but copying answers is strictly forbidden.

## Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Exercise 1.3 adapted; 10 points) The formal definition of a DFA $M$ is $(\{q_1, q_2, q_3, q_4, q_5\}, \{\mathtt{a}, \mathtt{b}\}, \delta, q_5, \{q_1\})$ where $\delta$ is given by the following table. Draw the state diagram of $M$ and give an intuitive characterization of the strings that $M$ accepts.

| | a | b |
|---|---|---|
| $q_1$ | $q_1$ | $q_2$ |
| $q_2$ | $q_1$ | $q_3$ |
| $q_3$ | $q_2$ | $q_4$ |
| $q_4$ | $q_3$ | $q_5$ |
| $q_5$ | $q_4$ | $q_5$ |

2. (Exercise 1.4; 20 points) Each of the following languages is the intersection of two simpler regular languages. In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in class (see also Footnote 3 in Page 46 of [Sipser 2006, 2013]) to give the state diagram of a DFA for the language given. In all parts, the alphabet is $\{\mathtt{a}, \mathtt{b}\}$.

   (a) $\{w \mid w$ has an even number of a's and one or two b's$\}$.

   (b) $\{w \mid w$ starts with an a and has at most two b's$\}$.

3. (Exercise 1.5; 20 points) Each of the following languages is the complement of a simpler regular language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts, the alphabet is $\{\mathtt{a}, \mathtt{b}\}$.

   (a) $\{w \mid w$ contains neither the substring ab nor ba$\}$.

(b) $\{w \mid w \text{ is any string not in } \mathtt{a}^*\mathtt{b}^*\}$. (Note: $\mathtt{a}^*\mathtt{b}^*$ is a regular expression denoting $\{\mathtt{a}\}^* \circ \{\mathtt{b}\}^*$.)

4. (Problem 1.36; 10 points) For any string $w = w_1 w_2 \cdots w_n$, the *reverse* of $w$, written $w^R$, is the string $w$ in reverse order, $w_n \cdots w_2 w_1$. For any language $A$, let $A^R = \{w^R \mid w \in A\}$. Show that, if $A$ is regular, so is $A^R$.

5. (Problem 1.37; 20 points) Let

$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

$\Sigma_3$ contains all size 3 columns of `0`s and `1`s. A string of symbols in $\Sigma_3$ gives three rows of `0`s and `1`s. Consider each row to be a binary number and let

$$B = \{w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top two rows}\}.$$

For example,

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in B, \text{but} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \notin B.$$

Show that $B$ is regular. (Hint: working with $B^R$ is easier. You may assume the result claimed in the previous problem (Problem 1.36).)

6. (20 points) Generalize the proof of Theorem 1.25 of [Sipser 2006, 2013] (Pages 45–47) to handle $A_1$ and $A_2$ with different alphabets.