Homework Assignment #4

Due Time/Date

This assignment is due 1:20PM Tuesday, April 11, 2023. Late submission will be penalized by 20% for each working day overdue.

Note

Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building 2, or put it on the instructor's desk before the class on the due date starts. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Problem 1.43; 10 points) An *all*-NFA M is a 5-tuple $(Q, \Sigma, \delta, q, F)$ that accepts $x \in \Sigma^*$ if every possible state that M could be after reading input x is a state from F. Note, in contrast, that an ordinary NFA accepts a string if *some* state among these possible states is an accept state. Prove that all-NFAs recognize the class of regular languages.

2. (Problem 1.66; 20 points) Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA and let h be a state of M called its "home". A *synchronizing sequence* for M and h is a string $s \in \Sigma^*$ where $\delta(q, s) = h$ for every $q \in Q$. Say that M is *synchronizable* if it has a synchronizing sequence for some state h. Prove that, if M is a k-state synchronizable DFA, then it has a synchronizing sequence of length at most k^3. (Note: $\delta(q, s)$ equals the state where M ends up, when M starts from state q and reads input s.)

3. (Problem 1.67; 20 points) We define the *avoids* operation for languages A and B to be

   $$A \textit{ avoids } B = \{w \mid w \in A \text{ and } w \text{ doesn't contain any string in } B \text{ as a substring}\}.$$

   Prove that the class of regular languages is closed under the *avoids* operation.

4. (Problem 1.64; 20 points) If A is any language, let $A_{\frac{1}{2}-}$ be the set of all first halves of strings in A so that

   $$A_{\frac{1}{2}-} = \{x \mid \text{ for some } y, |x| = |y| \text{ and } xy \in A\}.$$

   Show that if A is regular, then so is $A_{\frac{1}{2}-}$.

5. (Problem 1.68; 20 points) Let $\Sigma = \{0, 1\}$.

   (a) Let $A = \{0^k x 0^k \mid k \geq 1 \text{ and } x \in \Sigma^*\}$. Show that A is regular.

(b) Let $B = \{0^k 1 x 0^k \mid k \geq 1 \text{ and } x \in \Sigma^*\}$. Show that B is not regular.

6. (Problem 1.51; 10 points) Prove that the language $\{w \in \{0,1\}^* \mid w \text{ is not a palindrome}\}$ is not regular. You may use the pumping lemma and the closedness of the class of regular languages under union, intersection, and complement. (Note: a *palindrome* is a string that reads the same forward and backward.)