

Homework Assignment #6

Due Time/Date

This assignment is due 2:20PM Tuesday, April 25, 2023; however, to be better prepared for the midterm exam on April 18, you should try to complete it before the exam. Late submission will be penalized by 20% for each working day overdue.

Note

Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building 2, or put it on the instructor's desk before the class on the due date starts. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2006, 2013] with probable adaptation.)

1. (Exercise 2.2; 10 points)
 - (a) Use the languages $A = \{a^n b^n c^m \mid m, n \geq 0\}$ and $B = \{a^m b^n c^n \mid m, n \geq 0\}$, together with the fact that $\{a^n b^n c^n \mid m, n \geq 0\}$ is not context free, to show that the class of context-free languages is not closed under intersection.
 - (b) Use the preceding part and DeMorgan's law to show that the class of context-free languages is not closed under complementation.
2. (Exercise 2.5; 20 points) Give informal descriptions and state diagrams of pushdown automata for the following languages. In all parts the alphabet Σ is $\{0, 1\}$.
 - (a) $\{w \mid \text{the length of } w \text{ is odd}\}$
 - (b) $\{w \mid w \text{ is a palindrome, that is, } w = w^R\}$
3. (Exercise 2.12; 10 points) Convert the following CFG to an equivalent PDA, using the procedure given in Theorem 2.20.

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T \times F \mid F \\ F &\rightarrow (E) \mid a \end{aligned}$$

4. (Problem 2.39; 20 points) Let $G = (V, \Sigma, R, \langle \text{STMT} \rangle)$ be the following grammar.

$$\begin{aligned} \langle \text{STMT} \rangle &\rightarrow \langle \text{ASSIGN} \rangle \mid \langle \text{IF-THEN} \rangle \mid \langle \text{IF-THEN-ELSE} \rangle \\ \langle \text{IF-THEN} \rangle &\rightarrow \text{if condition then } \langle \text{STMT} \rangle \\ \langle \text{IF-THEN-ELSE} \rangle &\rightarrow \text{if condition then } \langle \text{STMT} \rangle \text{ else } \langle \text{STMT} \rangle \\ \langle \text{ASSIG} \rangle &\rightarrow \text{a} := 1 \end{aligned}$$

$$\Sigma = \{\text{if, condition, then, else, a} := 1\}$$

$$V = \{\langle \text{STMT} \rangle, \langle \text{IF-THEN} \rangle, \langle \text{IF-THEN-ELSE} \rangle, \langle \text{ASSIG} \rangle\}$$

G is a *natural-looking* grammar for a fragment of a programming language, but G is ambiguous.

- (a) Show that G is ambiguous.
 - (b) Give a new unambiguous grammar for the same language.
5. (Problem 2.56; 20 points) If A and B are languages, define $A \diamond B = \{xy \mid x \in A \text{ and } y \in B \text{ and } |x| = |y|\}$. Show that, if A and B are regular, then $A \diamond B$ is context free.
 6. (Problem 2.43; 10 points) Let A be the language of all palindromes over $\{0, 1\}$ with equal numbers of 0s and 1s. Prove, using the pumping lemma, that A is not context free.
 7. (Problem 2.45; 10 points) Let $F = \{a^i b^j \mid i = kj \text{ for some positive integer } k\}$. Prove that F is not context free.