Homework Assignment #9

Due Time/Date

This assignment is due 2:20PM Tuesday, May 23, 2023. Late submission will be penalized by 20% for each working day overdue.

Note

Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building 2, or put it on the instructor's desk before the class on the due date starts. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Exercise 4.9; 10 points) Review the way that we define sets to be of the same size in Definition 4.12. Show that "are of the same size" is an equivalence relation.

2. (Problem 4.12; 10 points) Let A be a Turing-recognizable language consisting of descriptions of Turing machines, $\{\langle M_1\rangle, \langle M_2\rangle, \ldots\}$, where every M_i is a decider. Prove that some decidable language D is not decided by any decider M_i whose description appears in A. (Hint: you may find it helpful to consider an enumerator for A.)

3. (Problem 4.16; 20 points) Let $PAL_{\text{DFA}} = \{\langle M\rangle \mid M \text{ is a DFA that accepts some palindrome}\}$. Show that PAL_{DFA} is decidable. (Hint: Theorems about CFLs are helpful here.)

4. (Problem 4.18; 20 points) A *useless state* in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.

5. (Problem 4.22; 10 points) Let A and B be two disjoint languages. Say that language C *separates* A and B if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that any two disjoint co-Turing-recognizable languages are separable by some decidable language.

6. (Exercise 5.1; 10 points) Show that EQ_{CFG} is undecidable.

7. (Exercise 5.4; 20 points) If A is reducible to B and B is a regular language, does that imply that A is a regular language? Why or why not?